Chapter 4

Design of Structural Controllability for Complex Network Architecture

Amitava Mukherjee  
*IBM India Private Limited, India*

Ayan Chatterjee  
*Jadavpur University, India*

Debayan Das  
*Jadavpur University, India*

Mrinal K. Naskar  
*Jadavpur University, India*

**ABSTRACT**

Networks are all-pervasive in nature. The complete structural controllability of a network and its robustness against unwanted link failures and perturbations are issues of immense concern. In this chapter, we propose a heuristic to determine the minimum number of driver nodes for complete structural control, with a reduced complexity. We also introduce a novel approach to address the vulnerability of the real-world complex networks, and enhance the robustness of the network, prior to an attack or failure. The simulation results reveal that dense and homogenous networks are easier to control with lesser driver nodes, and are more robust, compared to sparse and inhomogeneous networks.

**1. INTRODUCTION**

With the recent advances in network sciences and technology, we are compelled to recognize that nothing happens in isolation. Most of the phenomena occurring around us are connected with an enormous number of other pieces of a complex universal puzzle (Tanner, 2004; Barabasi, 2002; Strogatz, 2001). Our biological existence, religious practices and the social world, vividly depict a pellucid story of interrelatedness. With the Internet dominating our lives in the 21st century, we are witnessing a revolution in the making. But, the underlying critical question is: are we ready to embrace the importance of networks around us?

We should learn to appreciate the importance of networks, as part of our daily lives. Recent developments indicate that networks will dominate the next hundreds of years, to a much greater extent than most people are even prepared to acknowledge (Barabasi, 2002; Strogatz, 2001) this fact.

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Complex networks are those real-world networks that are characterized by irregular non-trivial topological features, dynamically evolving with time (Strogatz, 2001; Dorogovtsev et al., 2003; Newman et al., 2006; Albert et al., 2002). Different neural networks in our body, various biological networks, the Internet, power-grid (Broder et al., 2000), World Wide Web, social networks etc., can effectively be modeled as complex networks (Strogatz, 2001; Dorogovtsev et al., 2003; Newman et al., 2006). Hence, complex networks form a crucial part of our daily lives (Dorogovtsev et al., 2003). Since the last decade, we have been witnessing a major surge of growing interest and research, with the main focus shifting from the analysis of small networks to that of systems with thousands or millions of nodes. Reductionism was the key driving force behind much of the research of the previous century (Barabasi, 2002). For decades, scientists and researchers have studied atoms and their constituents to understand the universe, molecules to comprehend life, individual gene to characterize and to examine human behavior. Now, relying on the results gathered from the research done in the previous century, we are close to knowing everything about the individual piece. We have successfully disassembled the nature by spending billions of research dollars. But now we are clueless as we run into the hard wall of complexity (Barabasi, 2002; Albert et al., 2002). Nature is not a well-designed puzzle with a unique solution. In complex systems, the components can reassemble in more ways than we can ever imagine. Nature exploits all-encompassing laws of self-organization, whose roots are still mysteries to us (Barabasi, 2002; Newman et al., 2006).

Networks with irregular and random topological behavior appear in almost all fields of modern science and economics. Focus of engineering analysis is based on controlling real-world systems. Similar analysis also holds for these networks. But existing classical control theory fails in case of complex networks for reasons described in later sections. Thus, the modern theory of structural controllability has gained prominence. Structural Control theory is the main tool in the analysis of large scale networks. Unlike classical control theory, structural control theory is based on graph theory. Dilation-free paths play a major role in this type of control and are discussed in details in this chapter. For any network, few certain nodes play the role of controllers. These nodes are known as the driver nodes. This is an implication of the famous Pareto Principle that the entire network can be structurally controlled by a smaller set of nodes (driver nodes). This chapter deals with the mathematical procedure of eliminating dilation in networks and thereby determining the driver nodes for more efficient control of complex network from the view point of structural controllability theory.

1.1. Motivation

Complete structural controllability of a complex network is a mandate for ubiquitous data flow through a complex network. By complete structural controllability it is meant that all nodes in a network either lie on some augmenting path controlled by a driver node or thenode itself is a driver node. Evaluating the structural controllability of any real-world complex network, requires the determination of maximum matching in the network (Chatterjee et al., 2013; Das et al., 2014). The notion of maximum matching has been elaborated in the later sections. The classical definition of controllability proposed by Kalman (Kalman, 1963), did not work for complex networks, mainly due to the basic fact that the real-world networks were directed, whereas the definition held for undirected networks. Also, in most practical cases, as the networks tended to expand, it was almost impossible to compute the rank of the controllability state matrix Q_r (Luenberger, 1979). Hence, the classical control theory did not serve our purpose for complex networks. Thus, to control efficiently the complex networks, Lin proposed the structural controllability theorem (Lin, 1974). To evaluate the structural controllability of a network, we required