On Foundations and Applications of the Paradigm of Granular Rough Computing

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ABSTRACT
Granular computing, initiated by Lotfi A. Zadeh, has acquired wide popularity as a tool for approximate reasoning, fusion of knowledge, cognitive computing. The need for formal methods of granulation, and means for computing with granules, has been addressed in this work by applying methods of rough mereology. Rough mereology is an extension of mereology taking as the primitive notion the notion of a part to a degree. Granules are formed as classes of objects which are a part to a given degree of a given object. In addition to an exposition of this mechanism of granulation, we point also to some applications like granular logics for approximate reasoning and classifiers built from granulated data sets.

Keywords: classification; fusion of knowledge; granules of knowledge; granulation of knowledge; granular systems; reasoning by granular logics; rough inclusions; rough mereology; rough sets; similarity

NOTIONS CENTRAL TO GRANULATION OF KNOWLEDGE
Granulation of knowledge is one of important aspects of the way in which the human brain works (see, e.g., Pal, 2004). A vast literature on emulating such aspects of the brain workings as granulation, fusion of knowledge, classification by means of neural networks, fuzzy logics, and so on, does witness the role the computer science society attaches to them. These aspects are studied with at least a twofold purpose. First, to get an insight into the processes of perception, concept formation, and reasoning in the brain; second, to transfer this knowledge into the realm of applications.

The emergence of ample paradigms like cognitive informatics and natural intelligence (see, e.g., Chan, Kisner, Wang & Miller, 2004; Kinsner, Zang, Wang, & Tsai, 2005; Patel, Patet, & Wang, 2003; Wang, 2007, Wang, Johnston, & Smith, 2002; Yao, Shi, Wang, & Kinsner, 2006) is due in large part to these
studies and emulations of vital aspects of the brain mechanism. This emergence is welcomed as it provides a forum for an intradisciplinary study of mechanisms of perception, cognition, and reasoning.

Our work is devoted to the aspect of granulation of knowledge with applications to synthesis (fusion) of knowledge from various sources, reasoning in uncertain situations by means of approximate logics and data classification: all these aspects vital for human intelligence.

In this section, we describe basic tools applied in our analysis of granulation. First, we discuss basic principles of rough set theory (Pawlak, 1982, 1991) in particular, the notion of an information system. Next, we give space to an introduction to mereology (Leśniewski, 1916) whose techniques are important in our development of granular computing. Then, similarity is discussed briefly as a bridge to an introduction to mereology (Polkowski, 2005, 2006; Polkowski & Skowron, 1997).

**Rough Set Analysis of Knowledge**

This approach consists in understanding knowledge as an ability to classify objects in a given universe into classes of objects identical with respect to a given collection of features (attributes). The formal framework for defining knowledge is an information system which is a pair \( I = (U, A) \), where \( U \) is a set of objects and \( A \) is a set of attributes, each \( a \in A \) being a mapping \( a: U \rightarrow V_a \) from \( U \) into the value set \( V_a \).

The collection of \( a \)-indiscernibility relations \( \text{IND}(I) = \{ \text{ind}(a): a \in A \} \), where \( \text{ind}(a) = \{ (u, v): u, v \in U, a(u) = a(v) \} \) describes indiscernibility of objects induced by the attribute \( a \), in the spirit of Leibniz's identity of indiscernibles principle.

For any subset \( B \subseteq A \), the relation \( \text{ind}(B) = \bigcap_{a \in B} \text{ind}(a) \) is the relation of \( B \)-indiscernibility. Its classes \( [u]_B = \{ v \in U: (u, v) \in \text{ind}(B) \} \) form \( B \)-elementary granules of knowledge. Unions of granules of the form \( [u]_B \) are \( B \)-granules of knowledge. Indiscernibility relations submit a set-theoretical tool for defining granules; from the logical point of view, indiscernibility classes can be regarded as meanings of certain formulas of a description logic.

Elementary formulas of description logic are descriptors (see, e.g., Pawlak, 1991), of the form \( (a = v) \), with the meaning defined as the set \( \{ u \in U: a(u) = v \} \). The meaning is extended to the set of formulas obtained from descriptors by means of connectives \( \lor, \land, \neg \), \( \rightarrow \) of propositional calculus, with recursively extended semantics:

\[
1. \quad [a \lor b] = [a] \lor [b] \\
2. \quad [a \land b] = [a] \land [b] \\
3. \quad [\neg a] = U \setminus [a] \\
4. \quad [a \rightarrow b] = [\neg a \lor b] \\
\]

For an indiscernibility class \( [u]_B \), the following equality holds,

\[
[u]_B = \bigwedge_{a \in B} (a = a(u)) 
\]

i.e., the meaning of the formula \( \bigwedge_{a \in B} (a = a(u)) \) is equal to the class \( [u]_B \). Thus, indiscernibility classes, hence granules, are definable in description logic. For this reason, they are also called exact sets (see Pawlak, 1991). Other sets of objects, which are not exact, are called rough. The notion of a granule and an exact set are thus interchangeable in case granules are defined in attribute-value based languages.

A particular case of an information system is a decision system; that is, a triple \( (U, A, d) \) in which \( d \) is a singled out attribute called the decision. Decision is an attribute whose value is set by an expert in its own language. In order to relate the conditional knowledge \( (U, \text{IND}(I)) \) to the world knowledge \( (U, \text{ind}(d)) \), decision rules are in use. A decision rule is an implication, in description logic:

\[
\bigwedge_{a \in B} (a = v_a) \rightarrow (d = w) 
\]

The decision rule is true, or certain, in case \( \bigwedge_{a \in B} (a = v_a) \subseteq [(d = w)] \); otherwise, the decision rule is partially true, or possible. A classifier is a set of decision rules. We return
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