Chance Hypotheses Testing

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ABSTRACT

This paper considers the problem of testing hypotheses about hybrid distributions which are models representing situations where impreciseness (explained through fuzzy measure) and randomness (explained through probability measure) coexist. A criterion similar to the Neyman-Pearson criterion is proposed for testing a simple chance null hypothesis against a simple chance alternative hypothesis. The suggested criterion has been applied for testing hypotheses about hybrid triangular Bernoulli distribution and hybrid Poisson distribution. Optimal properties of the resulting tests have also been investigated.

KEYWORDS
Chance Distribution, Chance Likelihood, Chance Measure, Hybrid Variable, Neyman-Pearson Chance Criterion

1. INTRODUCTION

Probability Theory is a highly popular subject which forms the basis of various branches of Statistics. This subject remains as a fascinating field of interest for hundreds of researchers. Contributions by researchers in different dimensions of the subject led to the development of tools meant for understanding and studying various aspects of uncertainty created by random phenomena. It is to be recalled that, as observed by Liu and Liu (2002) almost in all experimental studies, uncertainty enters into the picture not only through randomness but also in the form of impreciseness (also known as fuzziness). Existing theoretical ideas and available tools in probability theory are not sufficient to handle such situations. Sensing this inadequacy, Liu and Liu (2002) developed a new branch of study called Credibility Theory to explore and understand the functioning of systems under imprecise situations.

Researchers working in Credibility theory study various aspects which are analogues of those available in Probability theory. For example, several credibility distributions are being developed. Their properties and applications are also investigated by them. It is to be mentioned that the credibility distributions can be compared with probability distributions which are nothing but stochastic cum mathematical representations of random situations arising in real life scenario. Equipossible, Triangular, Trapezoidal, Normal and Exponential credibility distributions are some examples of credibility distributions which are mathematical representations explaining different forms of imprecise situations. For more details on such distributions one can refer to Liu (2008). Realizing the need for studies on credibility theory which are parallel to similar studies under Statistical Theory of Testing

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of Hypotheses, Sampath and Ramya (2015) have initiated studies on developing test procedures for testing hypotheses about credibility distributions. It is important to mention that Sampath and Ramya (2015) introduced a criterion called membership ratio criterion for testing credibility hypothesis and applied the same for testing credibility hypothesis about fuzzy triangular distribution. Using the suggested criterion, Sampath and Ramya (2014) made a study about credibility hypothesis testing of the expectation of fuzzy normal distribution. A study on the existence of optimal tests for testing credibility hypothesis about fuzzy exponential distribution has been done by Sampath and Ramya (2015). In this connection, it is to be recalled that prior to Liu’s Credibility theory, Zadeh (1965) introduced Fuzzy Set Theory for understanding imprecise situations. Studies on Testing of hypotheses about imprecise quantities based on the approach of Zadeh (1965) have been made. For example, Arnold (1996) presented an approach for testing fuzzily formulated hypotheses with crisp data, Filzmoser and Viertl (2004) presented an approach for statistical testing by using the characterizing function of the fuzzy $p$-values and Torabi and Mirhosseini (2009) gave the Neyman-Pearson lemma to test fuzzy hypotheses. The concept of fuzzy test statistic using $\alpha$-cuts was given by Arefi and Taheri (2011). Testing fuzzy hypothesis with fuzzy data has been discussed by many researchers including Grzegorzewski (2002), Kruse and Meyer (1987) and Taheri and Behboodian (2006).

As observed earlier, Probability and credibility theories are meant for random and imprecise situations respectively. However neither of them can handle situations involving both impreciseness as well as randomness on its own. For example, think of a queuing system where it is assumed that inter arrival time follows exponential (probability) distribution. In this case, if one is interested in determining the probability related to number of arrivals in a given period of time then Poisson (probability) distribution can be used if the parameter involved in the exponential distribution is precisely known. However, in many situations such values remain unknown on several occasions. In the absence of such values, properties of such parametric quantities can be explained through credibility distributions constructed with the help of prior experience or subject experts’ guidance. This is an example where one can see in the model the presence of both randomness and impreciseness. To deal with such situations, Liu (2008) developed a theory known as Chance Theory which is a hybridization of probability and credibility theories. A brief description on chance theory and some related definitions are given in Section 2.

Distributions defined on chance spaces are called Hybrid Distributions. The concept of Hybrid distributions is nothing but the chance space analogue of the notions of probability distributions defined on probability space and credibility distributions defined on credibility space. More on hybrid distributions will be discussed in the coming sections of this paper. Several researchers have made their contributions in the theoretical developments and application aspects of chance theory. Sampath (2012), Sampath and Deepa (2014), and Sampath, Lalitha and Ramya (2015) have considered hybrid binomial, hybrid Poisson and hybrid normal distributions where the hybrid distributions are modeled by treating the parameters appearing in the corresponding probability distributions as fuzzy variables. Sampath (2009) considered an application of the tools available in Chance theory in designing acceptance sampling plans for use in Quality Control systems where impreciseness and randomness coexist in the system being studied. Sampath and Deepa (2013) designed acceptance sampling plans for situations involving both randomness and fuzziness using hybrid binomial and hybrid Poisson distributions. Sampath et al. (2015) considered designing of acceptance sampling for variables using hybrid normal distribution.

The above mentioned works are related to applications of Chance theory in developing quality control systems. Existence of situations which can be described by hybrid distributions warrant studies on different directions. In the process of deciding a suitable hybrid distribution one has to identify the underlying distribution and assign suitable values for the parameters involved in such distributions. When more than one set of values is available for the parametric quantities the practitioner is left with the task of choosing the most appropriate one. This requires developing a theory parallel to statistical theory of testing of hypotheses and testing of credibility hypothesis. It is pertinent to note that so
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