Chapter 43

Wavelets with Application in Image Compression

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**ABSTRACT**

This chapter focuses mainly on different wavelet transform algorithms as Burt’s Pyramid, Mallat’s Pyramidal Algorithm, Feauveau’s non dyadic structure and its application in Image compression. This chapter focuses on mathematical concepts involved in wavelet transform like convolution, scaling function, wavelet function, Multiresolution analysis, inner product etc., and how these mathematical concepts are liked to image transform application. This chapter gives an idea towards wavelets and wavelet transforms. Image compression based on wavelet transform consists of transform, quantization and encoding. Basic focus is not only on transform step, selection of particular wavelet, wavelets involved in new standard of image compression but also on quantization and encoding, Huffman code, run length code. Difference in between JPEG and JPEG2000, Quantization and sampling, wavelet function and wavelet transform are also given. This chapter is also giving some basic idea of MATLAB to assist readers in understanding MATLAB Programming in terms of image processing.

**INTRODUCTION**

**History of Wavelets**

Wavelet is a mathematical tool having a wide application in many areas. Wavelet transform allow time frequency localization. It was firstly introduced as Haar function in 1909 by Hungarian mathematician named Alfred Haar which consists of short positive pulse followed by short negative pulse. In 1930 English Mathematician Jhon Littlewood and R.E.A.C. Paley developed method of creating a signal well

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localized in frequency and relatively well localized in time. In 1946 Gabor Transform having greatest possible localization in time and frequency developed by a British Hungarian physicist Dennis Gabor. But new milestone was held by Morlet, who was an engineer developed his own way of analysing seismic signals by creating component localized in space known as “wavelet of constant shape” later named as “Morlet wavelets”. He proceed his work with Alex Grossmann, a physicist to confirm that waves could be reconstructed from its wavelet decompositions also wavelet transforms turned out better than Fourier transform. Morlet and Grossmann firstly introduced word “wavelet” in their paper published in 1984. Later Mayer discovered orthogonal wavelets. In 1986, Stephane Mallat, a former student of Mayer linked wavelet theory with subband coding and quadrature mirror filter. In 1987 Indrid Daubechies discovered a whole new class of wavelet which were orthogonal and without jumps, smooth wavelet which become an important tool in signal processing area used to break up digital data into contribution of various scales (Soman, Ramchandran & Resmi, 2011, p 1-15).


**PRELIMINARIES**

**Fourier Transform**

If \( \hat{\psi} \) represents Fourier transform given by

\[
\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t)e^{-i\omega t} \, dt.
\]

where \( \omega \) frequency and \( t \) is time parameter and inverse Fourier transform can be given by (Soman, Ramchandran & Resmi, 2011, p 33-48)

\[
f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \hat{f}(\omega)e^{-i\omega t} \, d\omega.
\]

**Wavelets**

Mathematically if \( \psi \in L^2(\mathbb{R}) \) and \( a \) is scaling and \( b \) is translation parameter satisfy following admissibility criterion (Chui, 1992, p 60-65; Soman, Ramchandran & Resmi, 2011, p 33-37)