Optimization Models for the Continuous Review Inventory System

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ABSTRACT

Businesses that carry inventory as a means of responding to customer demand must decide when to place inventory orders and how much to order. A model for selecting an optimal order quantity and reorder point in the continuous review system is implemented in spreadsheet software. Historical data on period demand and lead time is processed by performing a statistical test to select a mixture distribution for lead time demand. This distribution is incorporated into an expected shortage calculation and cost model where shortages incur lost sales and/or goodwill costs. The lead time demand distribution selection is evaluated through experiments. In examples, employing an incorrect distribution leads to expenditures between one and eighteen percent higher than those experienced with the correct model. A mismatch of the backorder or lost sales model with the correct underlying assumptions leads to expenses between four and thirteen percent larger than necessary in an example problem.

KEYWORDS
Continuous Review, Gamma Distribution, Inventory, Lead Time Demand, Mixture Distribution, Normal Distribution, Reorder Point, Spreadsheet Model, Uncertainty

1. INTRODUCTION

Inventory consists of finished goods or raw materials held in anticipation of future demand from a customer or production process. Decisions regarding the quantity and timing of inventory purchases affect the ability of a business to remain competitive by controlling costs and increasing responsiveness. Inventory purchases can be planned and executed to balance the costs of holding excess inventory with the costs of frequent orders and potential loss of customer sales.

This paper describes optimization models for determining inventory policies in a single-product, single-buyer setting that employ a mixture distribution for modeling lead time demand (LTD). This setting is appropriate to consider in cases where the buyer and supplier of an inventory item are decentralized, i.e. they are not members of the same organization or cannot feasibly coordinate their policies. The latter may apply to small businesses that do not have significant bargaining power to negotiate coordination incentives with a supplier and/or purchase only one item from a supplier. LTD is the number of units ordered by customers between the time an inventory replenishment order is placed and the time the goods are received from the supplier.

For items replenished on a regular basis, inventory is often managed using two policy values: a reorder point \( (R) \), which determines the inventory level remaining when placing an order, and the order quantity \( (Q) \). The inventory control policy that uses the \( (Q, R) \) values is a continuous review system. The reorder point is enough to cover the average demand over the lead time from the supplier, plus a safety stock \( (SS) \) level that protects against running out of inventory while waiting for delivery.
of an order. Thus, when we develop the spreadsheet optimization model for the buyer’s inventory management problem, \( Q \) and \( R \) will be the decision variables chosen to minimize total inventory costs.

In the method presented in this article, LTD conditional on each possible value of lead time is modeled separately, leading to a very accurate representation of lead time and demand uncertainty in the inventory management problem. The appropriate continuous distribution for demand per unit time is selected by performing a statistical test on empirical data, so the manager is not required to select a theoretical probability distribution for lead time or period demand. The model can be implemented in spreadsheet software, such as Microsoft Excel®. Accurate representation of LTD leads to improved inventory policies for \( Q \) and \( R \).

The next section provides a review of prior research concerning LTD distributions and continuous review inventory systems. This article is closely related to prior work in two areas: 1) use of mixture distributions for modeling LTD (Eppen & Martin, 1988; Tyworth, 1992; Keaton, 1995; Cobb, 2013a; Cobb, Rumí, & Salmerón, 2013; Wu & Tsai, 2001); and 2) optimal order quantity and reorder point determination in the continuous review inventory system (Tyworth, Guo, & Ganeshan, 1996; Tyworth & Zeng, 1998; Namit & Chen, 1999; Tyworth & Gaheshan, 2000; Winston & Albright, 2011; Cobb, 2013b). In the next section, the method presented in this paper will be further contrasted with prior research. The primary contributions of the method presented here are to select a distribution that best fits empirical demand data and incorporate one of two cost models based on the appropriate assumptions for a business.

After the literature review, the paper is structured as follows. Section 3 describes important notation, the probability model for LTD, and the buyer cost function. The modeling approach used to obtain the LTD distribution and expected shortage calculations is provided in Section 4. The details of the optimization model are given in Section 5. Examples to illustrate the performance of the model in selecting a distribution and providing effective inventory policies based on sample data are detailed in Section 6. The last section concludes the paper and provides directions for future research.

2. LITERATURE REVIEW

This literature review first addresses prior work in establishing a probability distribution for lead time demand, then reviews the modeling techniques that are most closely related to the approach described in this paper. Bartolacci, LeBlanc, Kayikci, & Grossman (2012) give an excellent overview of optimization models suggested for many different logistics applications, and Acharya, Gonzales, Eksioglu, & Arora (2014) describe an Excel implementation of a decision support tool in Excel for management of a supply chain.

2.1. Lead Time Demand Distributions

Jointly determining \( Q \) and \( R \) in the continuous inventory system requires the assignment of a probability distribution for LTD. The normal distribution has traditionally been used to model demand (Tyworth & O’Neill, 1997); however, a normal approximation with the identical mean and variance of the actual LTD distribution may not fit the shape of the true distribution (Lau & Lau, 2003; Lin, 2008). Providing alternatives to the normal approximation to the LTD distribution has been a popular subject of past research in inventory management.

Moment matching solutions have been developed that involve calculating the mean and variance of LTD from the mean and variance of demand per unit time (DPUT) and lead time (LT), then assigning a standard PDF as a model for the LTD distribution (Bagchi, Hayya, & Chu, 1986). For example, reorder points for commercial aircraft parts are calculated by Taylor (1961) by employing the negative binominal distribution. Standard probability distributions developed from Air Force data are used by Mitchell, Rappold, & Faulkner (1983) to calculate inventory reorder point values. Other standard PDFs that have been used to lead times as part of an LTD model include the lognormal, gamma, normal, and Weibull distributions (Bagchi et al., 1986). Department of Defense agencies are
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