Chapter 2
Neutrosophic Sets and Logic

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ABSTRACT

Neutrosophic sets and Logic plays a significant role in approximation theory. It is a generalization of fuzzy sets and intuitionistic fuzzy set. Neutrosophic set is based on the neutrosophic philosophy in which every idea Z has opposite denoted as anti(Z) and its neutral which is denoted as neut(Z). This is the main feature of neutrosophic sets and logic. This chapter is about the basic concepts of neutrosophic sets as well as some of their hybrid structures. This chapter starts with the introduction of fuzzy sets and intuitionistic fuzzy sets respectively. The notions of neutrosophic set are defined and studied their basic properties in this chapter. Then we studied neutrosophic crisp sets and their associated properties and notions. Moreover, interval valued neutrosophic sets are studied with some of their properties. Finally, we presented some applications of neutrosophic sets in the real world problems.

INTRODUCTION

The data in real life problems like engineering, social, economic, computer, decision making, medical diagnosis etc. are often uncertain and imprecise. This type of data is not necessarily crisp, precise and deterministic nature because of their fuzziness and vagueness. To handle this kind of data, (Zadeh, 1965) introduced fuzzy set sets. Several types of approaches have been proposed which is based on fuzzy sets such as (interval valued fuzzy sets, 1986), (intuitionistic fuzzy sets, 1986), and so on. Researchers throughout the world have been successfully applied fuzzy sets in several areas like signal processing, knowledge representation, decision making, stock markets, pattern recognition, control, data mining, artificial intelligence etc.

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Atanassov (1986) observed that there is some kind of uncertainty in the data which is not handled by fuzzy sets. Therefore, intuitionistic fuzzy sets were proposed (Atanassov, 1986), which became the generalization of fuzzy sets by inserting the non-membership degree to fuzzy sets. An intuitionistic fuzzy set has a membership function as well as a non-membership function. Intuitionistic fuzzy sets define more beautifully the fuzzy objects of the real world. A huge amount of research study has been conducted on intuitionistic fuzzy sets from different aspects. Intuitionistic fuzzy sets have been successfully applied in several fields such as modeling imprecision, decision making problems, pattern recognition, economics, computational intelligence, medical diagnosis and so on.

Smarandache (1995), coined the theory of neutrosophic sets and logic under the neutrosophy which is a new branch of philosophy that study the origin, nature, and scope of neutralities as well as their interactions with ideational spectra. A neutrosophic set can be characterized by a truth membership function \( T \), an indeterminacy membership function \( I \) and falsity membership function \( F \). Neutrosophic set is the generalization of fuzzy sets, intuitionistic fuzzy sets, paraconsistent set etc. Neutrosophic sets can treat uncertain, inconsistent, incomplete, indeterminate and false information. The neutrosophic sets and their related set theoretic operators need to be specified from scientific or engineering point of view. Indeterminacy are quantified explicitly in neutrosophic sets and T, I, and F operators are complementally independent which is very significant in several applications such as information fusion, physics, computer, networking, decision making, information theory etc.

In this chapter, we present the notions of neutrosophic sets and logic. In section 1, we presented a brief introduction. In section 2, we studied neutrosophic sets with some of their basic properties. In the next section 3, the hybrid structure neutrosophic crisp sets and their associated properties and notions have been studied. In section 4, interval valued neutrosophic sets have been studied. Section 5 is about to study some practical life applications of neutrosophic sets.

**NEUTROSOPHIC SET, SIMILARITY MEASURES, NEUTROSOPHIC NORMS**

In this section the notions of neutrosophic sets, some similarity measures of neutrosophic sets, neutrosophic norms respectively.

**Neutrosophic Set**

In this subsection the neutrosophic set is presented with their basic properties and notions with illustrative examples.

**Definition 2.1.1**

Let \( X \) be a universe of discourse and a neutrosophic set \( A \) on \( X \) is defined as

\[
A = \left\{ \left( x, T_A(x), I_A(x), F_A(x) \right), x \in X \right\}
\]

where \( T, I, F: X \rightarrow ]0,1[ \) and

\[
\sum_{x \in X} T_A(x) + I_A(x) + F_A(x) = 1
\]