Chapter 3
Interval–Valued Intuitionistic Fuzzy Partition Matrices

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ABSTRACT
If in an interval-valued intuitionistic fuzzy matrix each element is again a smaller interval-valued intuitionistic fuzzy matrix then the interval-valued intuitionistic fuzzy matrix is called interval-valued intuitionistic fuzzy partition matrix (IVIFPMs). In this paper, the concept of interval-valued intuitionistic fuzzy partition matrices (IVIFPMs) are introduced and defined different types of interval-valued intuitionistic fuzzy partition matrices (IVIFPMs). The operations like direct sum, Kronecker sum, Kronecker product of interval-valued intuitionistic fuzzy matrices are presented and shown that their resultant matrices are also interval-valued intuitionistic fuzzy partition matrices (IVIFPMs).

INTRODUCTION
Atanassov (1986) introduced the concept of intuitionistic fuzzy sets (IFSs), which is a generalization of fuzzy subsets. Later on much research works have done with this concept by Atanasov and others. The term fuzzy matrix has important role in fuzzy algebra. For definition of fuzzy matrix we follow the definition of Duobois and Prade (1980), i.e. a matrix with fuzzy member as its element. This class of fuzzy matrices consist of applicable matrices which can model uncertain aspects and the works on them are limited. Some of the most interesting works on these matrices done by Tan(2005). Xin (1992). Thomson (1977) defined convergence of a square fuzzy matrix. Pal and Shyamal (2002) introduced two new operators on fuzzy matrices and shown several properties of them. By the concept of IFSs, first time Pal (2001) introduced intuitionistic fuzzy determinant. Latter on Pal et al. (2002) introduced intuitionistic fuzzy matrices (IFMs) and distance between intuitionistic fuzzy matrices. Bhowmik and Pal (2008) presented some results on intuitionistic fuzzy matrices, intuitionistic circulant fuzzy matrices and generalized intuitionistic fuzzy matrices. Adak et al. investigated some interesting properties of interval-valued intuitionistic fuzzy sets and matrices (2011, 2012)
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The general rectangular or square array of the numbers are known as matrix and if the elements are interval-valued intuitionistic fuzzy then the matrix is called interval-valued intuitionistic fuzzy matrix. If we delete some rows or some columns or both or neither then the interval-valued intuitionistic fuzzy matrix is called interval-valued intuitionistic fuzzy submatrix. The concept of non-empty subset in set theory and the principle of combination are used for the construction and calculation of the number interval-valued intuitionistic fuzzy submatrices of a given interval-valued intuitionistic fuzzy matrix.

Again, if an interval-valued intuitionistic fuzzy matrix is divided or partitioned into smaller interval-valued intuitionistic fuzzy matrices called cells or blocks with consecutive rows and columns by drawing dotted horizontal lines of full width between rows and vertical lines of full height between columns, then the interval-valued intuitionistic fuzzy matrix is called interval-valued intuitionistic fuzzy block matrix. There are lots of advantages noted in partitioning an interval-valued intuitionistic fuzzy matrix $A$ into blocks or cells. It simplifies the writing or printing of an IFM $A$ in compact form and thus save space. It exhibits some smaller structure of $A$. It also simplifies computation.

The structure of this chapter is organized as follows. In Section 2, the preliminaries and some definitions are given. In Section 3, different kinds of interval-valued intuitionistic fuzzy submatrices and block matrices are given. Section 4 deals with direct sum, Kronecker sum and Kronecker product of interval-valued intuitionistic fuzzy block matrix. Finally, at the end of this paper a conclusion is given.

PRELIMINARIES

In this section, the concept of interval arithmetics are recalled. Let $[I]$ be the set of all closed subintervals of the interval $[0,1]$. An interval on $[I]$, say $\overline{a}$, is a closed subinterval of $[I]$ i.e., $\overline{a} = [a^-, a^+]$ where $a^-$ and $a^+$ are lower and upper limits of $\overline{a}$ respectively and satisfy the condition $0 \leq a^- \leq a^+ \leq 1$. For any two interval $\overline{a}$ and $\overline{b}$ where $\overline{a} = [a^-, a^+]$ and $\overline{b} = [b^-, b^+]$ then:

1. $\overline{a} = \overline{b} \iff a^- = b^-, a^+ = b^+$,
2. $\overline{a} \leq \overline{b} \iff a^- \leq b^-, a^+ \leq b^+$ and
3. $\overline{a} < \overline{b} \iff a^- < b^-, a^+ < b^+$ and $\overline{a} \neq \overline{b}$.

Definition (Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs)): An IVIFS $A$ over $X$ (universe of discourse) is an object having the form

$$A = \{(x, M_A(x), N_A(x)) \mid x \in X\},$$

where $M_A(x): X \rightarrow [I]$ and $N_A(x): X \rightarrow [I]$.

The intervals $M_A(x)$ and $N_A(x)$ denote the intervals of the degree of membership and degree of non-membership of the element $x$ to the set $A$, where

$$M_A(x) = [M_{AL}(x), M_{AU}(x)]$$

and

$$N_A(x) = [N_{AL}(x), N_{AU}(x)].$$