Chapter 12

Decision Making by Using
Intuitionistic Fuzzy Rough Set

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ABSTRACT

This chapter begins with a brief introduction of the theory of rough set. Rough set is an intelligent technique for handling uncertainty aspect in the data. This theory has been hybridized by combining with many other mathematical theories. In recent years, much decision making on rough set theory has been extended by embedding the ideas of fuzzy sets, intuitionistic fuzzy sets and soft sets. In this chapter, the notions of fuzzy rough set and intuitionistic fuzzy rough (IFR) sets are defined, and its properties are studied. Thereafter rough set on two universal sets has been studied. In addition, intuitionistic fuzzy rough set on two universal sets has been extensively studied. Furthermore, we would like to give an application, which shows that intuitionistic fuzzy rough set on two universal sets can be successfully applied to decision making problems.

INTRODUCTION

In our day to day life, most of the situations and associated data we come across are of not certain due to presence of uncertainties in the physical world. Uncertainty exists almost everywhere, even in the most idealized circumstances. Data set from real-life applications like medical, business, economics, etc. is not crisp and has uncertainties. In addition, uncertainty is an attribute of information and usually decision-relevant information is uncertain and imprecise. It is imperative to model these day to day problems involving uncertainties mathematically.

To this end, theories like probability theory, fuzzy set theory, intuitionistic fuzzy set theory, rough set theory, etc. to deal with problems involving uncertainties. However each theory has their limitations. In fact, the inadequacy of the parameterization tool in these theories does not allow them to handle vagueness properly.

In 1999, Molodtsov introduced the concept of soft sets and established the fundamental results of the new theory. Soft set theory somehow is free from the difficulties present in fuzzy set theory, rough
set theory, probability theory, etc. In 2002, P.K. Maji studied the theory of soft sets and exhibited some results of the soft set in decision making tasks. Further, researchers redefined complement of a soft set and showed that the axioms of exclusion and contradiction are satisfied by the soft sets also.

This chapter begins with a brief introduction of soft set and then describes generalizations of it. This chapter endeavors to forge a connection between fuzzy set and soft set and depicts a new model fuzzy soft set to address the challenges of vagueness and impreciseness. Furthermore, an attempt has been made to hybridize soft set with intuitionistic fuzzy set. We present a brief overview on intuitionistic fuzzy sets, which cuts across some definitions, operations, algebra, modal operators and normalization on intuitionistic fuzzy set.

FOUNDATIONS OF ROUGH COMPUTING

At the present age of internet, a huge repository of the data is available across various domains. Therefore, it is very hard to extract useful information from voluminous data available in the universe. So, information retrieval and knowledge representation has become one of the most popular areas of recent research. Information retrieval and acquisition of knowledge is one of the important components of an information system. But, the real challenge lies in converting voluminous data into knowledge and to use this knowledge to make proper decisions. In order to transform the processed data into useful information and knowledge, there is a need of new techniques and tools. Rough set theory developed by Pawlak (1982) used to process uncertain and incomplete information is a tool to the above mentioned problem. One of its strength is the attribute dependencies, their significance among inconsistent data. At the same time, it does not need any preliminary or additional information about the data. Therefore, it classifies imprecise, uncertain or incomplete information expressed in terms of data acquired from experience.

Rough Sets

In this section we recall the definitions of basic rough set theory developed by Pawlak (1991). Let \( U \) be a finite nonempty set called the universe. Suppose \( R \subseteq (U \times U) \) is an equivalence relation on \( U \). The equivalence relation \( R \) partitions the set \( U \) into disjoint subsets. Elements of same equivalence class are said to be indistinguishable. Equivalence classes induced by \( R \) are called elementary concepts. Every union of elementary concepts is called a definable set. The empty set is considered to be a definable set, thus all the definable sets form a Boolean algebra and \((U,R)\) is called an approximation space. Given a target set \( X \), we can characterize \( X \) by a pair of lower and upper approximations. We associate two subsets \( \overline{RX} \) and \( \underline{RX} \) called the R-lower and R-upper approximations of \( X \) respectively and are given by

\[
\overline{RX} = \bigcup \{Y \in U \mid R : Y \subseteq X\}
\]

\[
\underline{RX} = \bigcup \{Y \in U \mid R : Y \cap X \neq \emptyset\}
\]

The \( R \)-boundary of \( X \), \( BN_R(X) \) is given by \( BN_R(X) = \overline{RX} - \underline{RX} \). We say \( X \) is rough with respect to \( R \) if and only if \( \overline{RX} \neq \underline{RX} \), equivalently \( BN_R(X) \neq \emptyset \). \( X \) is said to be \( R \)-definable if and only if \( \overline{RX} = \underline{RX} \) or \( BN_R(X) = \emptyset \). So, a set is rough with respect to \( R \) if and only if it is not \( R \)- definable.