Chapter 8
Making Links Between Solutions to an Unstructured Problem: The Role of Pre-Written, Designed Student Responses

Sheila Evans
University of Nottingham, UK

ABSTRACT
In the study described here, teaching resources have been developed to provide students with explicit opportunities to link invariant properties across a range of different solution strategies, and make comparative judgments about the same solutions. After tackling an unstructured problem, students complete, compare and critique pre-designed student responses to the same problem. The framework used to analyze the data focuses on the types of links students may make between responses. The findings indicate students made varied links when completing them. The outcome of these links appeared to be influenced by how students perceived the representation being completed. Students made further assorted links that focused on invariant properties and the comparative validity of the completed responses.

INTRODUCTION
When solving an unstructured problem, novice problem solvers are not readily reminded of similarly structured problems from the past and can find it challenging to re-cycle ‘old’ knowledge in new situations (Gick & Holyoak, 1983). They often set vague, unstructured goals or their goals are flawed (Juwah et al., 2004). Instead, they rely on naïve, inefficient strategies such as ‘trial and improvement’ (Evans & Swan, 2014). These strategies they pursue relentlessly, without pausing to review their validity or consider alternative strategies (Schoenfeld, 1992). Failure to effectively monitor their emerging solution may be because they are uncertain of the criteria to judge the quality of their work (Bell, Philips, Shannon, & Swan, 1997), other than checking the correctness of the answer. Or when monitoring, they simply cannot think of a replacement strategy. On the other hand, expert problem solvers recognize structural similarities between a new problem-situation and past problems and are able to retrieve relevant knowledge to
Making Links Between Solutions to an Unstructured Problem

construct effective strategies. They spend time setting hierarchical goals and as their solution unfolds they carefully monitor and regulate their progress against their goals (Schunk & Zimmerman, 2006).

This contrasting behavior can be for a number of reasons. It is widely recognized that insufficient original learning of mathematical concepts can prevent students transferring knowledge to new situations (e.g. Lee & Pennington, 1993). In particular, when solving an unstructured problem, lack of knowledge of how a concept can be represented in different ways (Rittle-Johnson & Star, 2007) together with inadequate metacognitive strategies (Schoenfeld, 1989) can restrict students’ capacity to flexibly and usefully adapt their original approach. The research indicates linking different solutions to the same problem can broaden students’ understanding of a specific problem-situation, and more generally develop their conceptual understandings and metacognitive strategies (Brousseau, 1997; Chazan & Ball, 1999; Lampert, 2001; Stein, Eagle, Smith, & Hughes, 2008). As Kaput (1989, pp. 179–180) proposed: “the cognitive linking of representations creates a whole that is more than the sum of its parts...it enables us to see complex ideas in a new way and apply them more effectively.” However, while there is much research into students’ working with multiple solutions to a problem (Thompson, 1994), there are only a few studies that explore the process of students’ linking these solutions (Wilmot, Schoenfeld, Wilson, Champney, & Zahner, 2011).

This chapter forms part of a design research study in which resources are specifically created to develop students’ capacity to solve unstructured problems. Central to this design intention is the provision of opportunities for students to link different solutions to a problem. As an illustrative focus for the chapter, data from one lesson is explored. The chapter begins by considering the research that shaped both the resources and the analytical framework. Then, to explicate the design researcher’s intentions, the resources are described in detailed. This is followed by an analysis of the data. The analysis is framed by questions such as ‘When do students make links between different solutions, ‘Why do students make links?’ and ‘What properties do students link?’. The findings are then discussed. Finally, the author reflects on how the materials could be refined to provide further opportunities for both students’ link-making and robust investigations into their link-making.

THEORETICAL BACKGROUND

This research supports the view that developing students’ general metacognitive strategies and understandings of concepts underlying unstructured problems are key to developing students’ capacity to solve unstructured problems. These two capacities are interdependent, with one supporting and developing because of the other. For example, in the midst of solving an unstructured problem students may ask themselves: ‘Can I think of a quicker method?’ If they perceive mathematics as a series of disconnected algorithms to be memorised then any changes are likely to remain firmly within the representations. Whereas another student, with a more ‘connected’ understanding of mathematics may consider changing to a different representation of the same underlying concept.

Many studies cite monitoring and regulating as key discriminators for problem solving success (e.g. Schoenfeld, 1992; Carlson & Bloom, 2005). Expert problem solvers routinely use metacognitive strategies by stepping back from the problem and asking themselves or their partner questions such as ‘Where is this strategy going?’, ‘Should it be so complicated?’, ‘Does this solution make sense?’ (e.g. Schoenfeld, 1992). Answers to which, may prompt a recalculation, or a change of direction to improve, for example, the solution’s appropriateness, elegance, efficiency or generalizability. These questions will vary accord-