Chapter 6
Dynamic Stability and Post-Critical Processes of Slender Auto-Parametric Systems

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ABSTRACT

High-rise structures exposed to a strong vertical component of an earthquake excitation are endangered by auto-parametric resonance effect. While in a sub-critical state, the vertical and horizontal response components are independent. Exceeding a certain limit causes the vertical response to lose stability and induces dominant horizontal response. This effect is presented using two mathematical models: (1) the non-linear lumped mass model; and (2) the one dimensional model with continuously distributed parameters. Analytical and numerical treatment of both leads to three different types of the response: (1) semi-trivial sub-critical state with zero horizontal response component; (2) post-critical state (auto-parametric resonance) with a periodic or attractor type chaotic character; and (3) breaking through a certain limit, the horizontal response exponentially rises and leads to a collapse. Special attention is paid to transition from a semi-trivial to post-critical state in case of time limited excitation period as it concerns the seismic processes.

INTRODUCTION

This chapter is devoted to dynamic analysis of slender structures (towers, masts, chimneys, bridges, etc.) under pure vertical excitation acting in the base. Dynamic behaviour of such structures is a widely discussed topic in earthquake engineering literature. However, authors are dealing predominantly with an influence of horizontal excitation components. On the other hand, a strong vertical excitation component, especially in the earthquake epicentre area, can be decisive. Joint effect of horizontal and vertical components is rarely taken into account because the widely used linear approach usually does not...
provide any interesting knowledge in such a case. In linear models, the vertical and horizontal response components are independent. If no horizontal excitation is taken into account, no horizontal response component is observed. The mutual interference of horizontal and vertical components of the response becomes visible only if the non-linear models are adopted. The corresponding non-linear model is then of an auto-parametric character. As a consequence, e.g., if the frequency of a vertical excitation in a structure foundation finds in a certain interval and its amplitude exceeds a certain limit, the vertical response component loses dynamic stability and dominant horizontal response component is generated — even if no horizontal excitation is assumed. This post-critical regime (auto-parametric resonance) follows from a non-linear interaction of vertical and horizontal response components that can lead to a failure of the structure. On the other hand, until the amplitude or frequency of excitation remain outside the critical values, the non-linear relations do not manifest themselves. The response remains in sub-critical linear regime and follows the stable semi-trivial (i.e., almost linear) solution.

Qualitatively different post-critical response types can be detected when sweeping throughout the frequency interval beyond the stability limit. Deterministic as well as chaotic response types were observed. In general, they can occur in a steady state. Quasi-periodic and completely undetermined regimes, including a possible energy transflux, occur between degrees of freedom.

Similar effects can be encountered in different branches. For instance, a railway car running on an imperfect track generating periodical motion can result in a stability loss and possible derailment of the car (although other causes of this effect are not negligible) (Náprstek, 2013). Another example is the capsizing of ships on a wavy surface due to a periodic change in the metacentric height of the vessel or due to non-linear coupling between heave-roll or pitch-roll motions (Nabergoj & Tondl, 1994; Nayfeh, Mook, & Marshall, 1973; Tondl, Ruijgrok, Verhulst, & Nabergoj, 2000).

The whole problem is solved with two simplified non-linear models. One is the lumped mass model, where the elastic console is modelled by lumping the mass at the top and relating the mass to the basement by a massless spring. The other model assumes uniformly distributed stiffness and mass in order to respect the whole eigen-frequency spectrum. Both models serve their particular purposes. The lumped mass model is simpler and allows better analytical treatment to get a more detailed insight into its internal structure. The continuous model offers greater flexibility with respect to sensitivity to individual eigen-frequencies.

The outline of the chapter is as follows. First, the mathematical models of the structure are defined in the form of Lagrange equations. Although the massless spring and continuous console bending is considered linear, the models comprise non-linear relation between displacement and rotation of the base point and the console bending. The next section presents theoretical formulation of the semi-trivial solution and stability limits for both models. The linear perturbation approach is used. The response and perturbations are assumed in the form of harmonic functions. Existence of the stationary solution is assessed. Influence of system parameters for both models is studied. This is followed by a section discussing various types of the post-critical response. The assessment is mostly based on numerical evaluations of the stability conditions for various parameters. The case when the stability condition is overstepped is examined. It is shown that the system is able to withstand a limited duration of excitation exceeding the bounds defined by the stability conditions. If the excitation is stopped, the system is able to return to a standstill. Conclusions are presented in the last section.