Chapter 15
Measuring Complexity of Chaotic Systems With Cybernetics Applications

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ABSTRACT

Measuring complexity of systems is very important in Cybernetics. An aging human heart has a lower complexity than that of a younger one indicating a higher risk of cardiovascular diseases, pseudo-random sequences used in secure information storage and transmission systems are designed to have high complexity (to resist malicious attacks), brain networks in schizophrenia patients have lower complexity than corresponding networks in a healthy human brain. Such systems are typically modeled as deterministic nonlinear (chaotic) system which is further corrupted with stochastic noise (Gaussian or uniform distribution). After briefly reviewing various complexity measures, this chapter explores characterizing the complexity of deterministic nonlinear chaotic systems (tent, logistic and Hénon maps, Lorenz and Rössler flows) using specific measures such as Lempel-Ziv complexity, Approximate Entropy and Effort-To-Compress. Practical applications to neuron firing model, intra-cranial pressure monitoring, and cardiac aging detection are indicated.

INTRODUCTION

Norbert Weiner defined cybernetics as the ‘science of control and communication in the animal and the machine’ in 1948. The famous anthropologist Gregory Bateson defined cybernetics as a highly mathematical discipline dealing with “problems of control, recursiveness, and information”. There are many parallel traditions of cybernetics, but one that they all have in common is that they all are interested in the study of forms and patterns occurring in physical, biological, economic and social systems with an aim to predict, measure, control, process and communicate. Cybernetics has emerged as a highly inter-
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disciplinary area of research which is increasingly developing a meta-disciplinary or trans-disciplinary language by which we may better understand the world around us, in order to enable us to regulate and modify it to our needs.

The intersection of cybernetics with complex systems is a primary concern, since complex systems are ubiquitous. Biological systems (e.g. Heart, Brain), ecological systems (populations of species), financial systems (markets), electrical and electronic systems (millions of diodes connected to form an integrated circuit), social networks, computational systems and communication networks (the internet is a connection of millions of computers) are just a few examples of complex systems. Many of these are modeled as deterministic (typically non-linear) or stochastic or a hybrid of the two (with noise invariably added, every real-world system has a stochastic component). It is of vital importance to study these systems to understand their organization, information transmission with or without feedback, information processing and computation, prediction and control of such systems for desired performance.

One of the important steps in modeling, designing and analyzing complex systems in cybernetics is to measure complexity of time series, measurements or observations of the system under study. The main objective of this chapter is to deal exclusively with measuring complexity of complex systems – especially those systems which exhibit complicated behavior and which have potential applications in cybernetics. Such systems are known to exhibit ‘Chaos’ (a technical term which we will define shortly). After studying complexity of chaotic systems, practical applications of measuring complexity will be demonstrated in three specific cybernetics applications – analyzing a neuron firing model, intra-cranial pressure monitoring, and cardiac aging detection. We shall conclude by pointing to future research directions.

COMPLEX SYSTEMS

A complex system is defined as one in which the individual components that make up the system are by themselves simple but which produces complex behavior due to varied interactions amongst themselves (Northrop, 2010). The study of complex systems has been nascent in the past and has come to prominence only in the last couple of decades. This has been made largely possible by the availability of high-end tools for high speed computation and analysis.

Complex systems possess some basic properties, listed below (Lloyd, 2001).

- They are composed of simple components.
- The interactions among components happen in a non-linear fashion.
- Control of the systems is not centralized.
- The systems show evolution and learning and adapt to improve themselves.

Some examples of naturally occurring complex systems include brain, immune system and respiratory system to name a few. A healthy human heart is known to produce complex beat-to-beat variations. Brain networks in schizophrenia patients have lower complexity than corresponding networks in a healthy human brain. In the physical world, complex systems are frequently encountered in fields like dynamics, information processing, cryptographic protocols, weather prediction, computation and study of population and evolution. For example, pseudo-random sequences used in secure information storage