ABSTRACT

The last decades are characterized by significant progress in the development and operational use of modern numerical hydrodynamic methods of the Earth’s weather and climate. This was made possible primarily due to modern understanding of the laws governing the basic physical and thermodynamic processes in the atmosphere and the emergence of more advanced mathematical models and effective methods of their implementation. In this chapter, we develop new numerical techniques used to solve the non-stationary problem of general circulation of the atmosphere with a prehistory and the problem of planetary weather forecast.

A BRIEF OVERVIEW

The existing prediction models are classified according to the type of meteorological processes, specifics of their mathematical realization of the forecast period, as well as the size of the territory for which they are formulated. Until 1980 the operational weather services of the developed countries have focused on models of global forecasting of synoptic processes, despite some evidence that the quality of such forecasts remained almost constant. The increased interest in global models is explained, first of all, by political reasons and that they allow significant simplifications of the model equations and the use of effective methods of their numerical solution.

To implement a global model with a local high resolution with limited computing resources requires the use of efficient numerical methods. We will measure efficiency as the time that a single processor takes for the integration of model equations for one hour of the model time at a predetermined level of atmospheric circulation reproduction error. Of course, it is desirable that the methods used have an internal parallelism permitting efficient implementation on modern parallel computers.
The accuracy and efficiency of the existing numerical methods depend on the dimension of the problem. Choosing the best solution method is a difficult task as the purpose (accuracy) and means (cost) are contradictory to each other. Despite the constant improvement of computers, problems of accuracy and efficiency of numerical methods for solving problems of the circulation of the atmosphere remain valid today.

Currently, there are widely known various modifications of approaches to the numerical solution of the Navier-Stokes equations that are the basis of hydrodynamic models of atmospheric circulation. They can be divided into the following groups:

- **Direct Methods**: These methods (mainly Galerkin methods and their modifications (Fletcher, 1984; Morton & Parrot, 1980; Giraldo, 2000; Moorthi, Higgins, 1993) are accurate, efficient and cost-effective in required computer memory. However, their convergence is largely influenced by the choice of test functions. That is why direct methods are found in the use of convection problems with free or periodic boundaries, for which analytical solution of the linear problem is known.

- **Semi-Analytic Methods**: This approach combines such approximate methods as the method of integral relations, method of the straights, the method of the truncated series, etc., in which the solution of a system of differential equations in partial derivatives is reduced to solving a system of ordinary differential equations. The use of these methods is limited by difficulties in solving a system of ordinary differential equations, since their properties are usually radically different from those of the corresponding partial differential equations (e.g., stiffness of equations when applying direct methods). In studies of recent years, a number of very useful properties and promising results are received for the use of spectroscopic methods in numerical weather prediction based on hemispherical or global baroclinic hydrodynamic models (Boer et al., 1984; Carpenter et al., 1990; Ducrocq, Bougeault, 1995; Geleyn, Hollingsworth, 1979; Hortal, Simmons, 1991; Jakob-Chien, Hack, Williamson, 1995; Taylor, Tribbia, Iskandarani, 1997; Thuburn, Yong, 2000). Its essence consists in an approximation of some meteorological quantities at a time by a finite segment of the series expansion for linearly independent analytic functions defined in the whole domain of integration, and the analytic representation of spatial derivatives included in the equation. At present, the spectral models are used for operational numerical weather prediction in Australia, Canada and developed in other countries.

- **Finite Element Method**: (Argyris, Mareczek, Scharpf, 1970; Barrett, Morton, 1981; Griffiths, Mitchel, 1979; Mitchel, Griffiths, 1980; Cullen, 1973; Lele, 1992; Steger, Warming, 1981). The main advantage of this method is that the ruling equations are solved in a Cartesian coordinate system with the use of computational grids associated with the boundaries geometry of the field solutions and using an approximation based on variation principles. The main disadvantage of the method of finite elements is their cost inefficiency.

- **Finite Difference Methods**: These methods are the most versatile and widely applicable. The first attempts to use finite difference schemes for solving hydrodynamic problems relate yet to the 1930s. The advent of computers contributed to the further rapid development of finite difference methods for solving Navier-Stokes equations and the development of efficient algorithms for the solution of difference equations approximating the differential equations of elliptic, parabolic