Fuzzy Greedy Search: A Deterministic Heuristic for Combinatorial Optimization

Kaveh Sheibani, ORLab Analytics, Vancouver, Canada

ABSTRACT

This paper presents mathematics of the so-called fuzzy greedy evaluation concept which can be integrated into approaches for hard combinatorial optimization problems. The proposed method evaluates objects in a way that combines fuzzy reasoning with a greedy mechanism, thereby exploiting a fuzzy solution space using greedy methods. Given that the greedy algorithms are computationally inexpensive compared to other more sophisticated methods for combinatorial optimization; this shows practical significance of using the proposed approach. The effectiveness and efficiency of the proposed method are demonstrated on permutation flow-shop scheduling as one of the most widely studied hard combinatorial optimization problems in the area of operational research and management science.

KEYWORDS

Combinatorial Optimization, Fuzzy Sets, Greedy Algorithms, Heuristics, Scheduling, Sequencing

INTRODUCTION

In recent decades, there has been a growth of interest in methods for finding optimal solutions to a class of problems called combinatorial optimization. The subject is very wide and many books and articles have been published on its various aspects. Typical examples of combinatorial optimization problems are the travelling salesman (Lawler, 1985; Gutin & Punnen, 2002; Gutin, 2013), variants of the assignment problem (Cela et al., 2014), scheduling problems (Pinedo et al., 2015), the set covering (Beasley, 1990; Boschetti & Maniezzo, 2015), and vehicle routing problems (Coelho & Laporte, 2015). Combinatorial problems are normally easy to describe but difficult to solve (Korte & Vygen, 2012). The foundations for the theory of computational complexity are to be found in Cook’s (1971) seminal paper. In his paper, Cook attempted to classify problems in practice as easy or hard. A problem is called easy if an algorithm can be developed which solves the problem to optimality in a polynomial-time (Du & Pardalos, 2005). A problem is called hard or intractable if such efficient algorithms do not exist or the solution cannot be found within a reasonable computational time bound.

Due to the practical importance of combinatorial optimization problems, many methods have been developed for solving them (Pulleyblank (2014); Martello & Ries (2015); Nishi et al., (2017)). These methods can be classified as either exact or approximate. Exact methods guarantee to find an optimal solution in a bounded amount of time. Of course, for those combinatorial optimization problems...
problems which belong to the class NP-hard (Garey & Johnson, 1979), exact methods need an exponential amount of time. Thus, approximation methods which usually called heuristics, are often considered to be the only practical tools available to solve hard combinatorial optimization problems.

This paper overviews a new idea, so-called fuzzy greedy evaluation concept for the integration of approaches for hard combinatorial optimization problems. The paper also provides a comprehensive discussion on mathematics of the proposed methodology. In this presentation, the proposed method can be seen as a generic heuristic for the integration of approaches to hard combinatorial optimization problems. Furthermore, the current work shows another application of the proposed method with different objectives in addition to the previously published literature on the same subject. For example, it shows that how different adaptations of the proposed method can be yielded to different results.

The paper consists of two main parts. The first part focuses on description of the theory and mathematics of the so-called fuzzy greedy evaluation concept. The second part demonstrates computational examples of the proposed concept for hard combinatorial optimization problems. The concluding remarks contain some suggestions for further research.

**Fuzzy Sets**

In many real-world problems, fuzzy sets allow us to represent vague concepts expressed in natural language. The membership function of a fuzzy set $A$ can be denoted by $\mu_A: X \rightarrow [0, 1]$. Each fuzzy set should be uniquely defined by one particular membership function. Consider a fuzzy set where membership function is defined in Equation (1). This is one of the general formulæ of a parameterized family of membership functions described in Klir and Yuan (1995):

$$\mu_A(x) = \frac{1}{1 + \rho(x - r)^2}$$  \hspace{1cm} (1)

This fuzzy set expresses, in a particular form, the general concept of a class of real numbers that are close to $r$. When the non-negative parameter $\rho$ increases, the graph of $\mu_A(x)$ becomes narrower. The function has the following properties: $\mu_r(r) = 1$ and $\mu_r(x) < 1$ for all $x \neq r$. More detailed discussions of fuzzy sets and their properties can be found in (Klir & Yuan, 1995; Zimmermann, 2001; Wang & Klir, 2013).

**Greedy Algorithms**

The greedy method is an approach to algorithm design which is particularly suited for solving certain optimization problems. Such algorithms make choices according to what appears to be the best choice at the moment, without any consideration of any previous choices or future consequences. Greedy algorithms do not necessarily yield optimal solutions, but for some problems they do.

The greedy paradigm is often used in solving combinatorial optimization problems (Cook et al., 1998). This phenomenon can be explained by the fact that, although it is widely assumed that greedy algorithms rarely produce optimal solutions, they often provide solutions that are significantly better than the worst case (Vince, 2002; Curtis, 2003; Bang-Jensen et al., 2004). Hence, a natural question to pose is: can we characterize the type of optimization problems for which greedy algorithms produce optimal solutions? In a sense, the theory of matroids (Whitney, 1935; Edmonds, 1971), and later greedoids (Korte & Lovasz, 1981; Korte & Lovasz, 1983; Korte et al., 1991) are useful in determining when the greedy method yields optimal solutions. Vince (2002) put greedy algorithms into a simple framework, which explains when they do not work for all linear objective functions. Curtis (2003) addressed principles for the classification of those greedy algorithms that do provide optimal solutions to optimization problems. In extreme cases, for certain cases of optimization problems, a greedy algorithm may produce the unique worst possible solution (Gutin & Yeo, 2002; Gutin et al., 2002). Bang-Jensen et al. (2004) characterized certain cases in which the greedy algorithm fails.
Related Content

A Fast and Deterministic Approach to a Near Optimal Solution for the p-Median Problem
[www.igi-global.com/article/fast-deterministic-approach-near-optimal/69175?camid=4v1a](www.igi-global.com/article/fast-deterministic-approach-near-optimal/69175?camid=4v1a)

Understanding the Realities of Contemporary Internationalization of the MNE: Regionalization vs. Globalization
[www.igi-global.com/chapter/understanding-realities-contemporary-internationalization-mne/63911?camid=4v1a](www.igi-global.com/chapter/understanding-realities-contemporary-internationalization-mne/63911?camid=4v1a)

Food Systems: How Can the Normative Practice Approach Help Toward a Just and Sustainable Food System?
[www.igi-global.com/chapter/food-systems/225310?camid=4v1a](www.igi-global.com/chapter/food-systems/225310?camid=4v1a)
Random Node Pair Sampling-Based Estimation of Average Path Lengths in Networks
www.igi-global.com/article/random-node-pair-sampling-based-estimation-of-average-path-lengths-in-networks/206243?camid=4v1a