Contiguity-Based Optimization Models for Political Redistricting Problems

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ABSTRACT

Political redistricting is a process used to redraw political boundaries based on a number of criteria that include population equality, minority representation, contiguity, and compactness. Redistricting plans can be difficult to draw manually and since the 1970s the use of computers in the creation of redistricting plans has increased dramatically. The purpose of this paper is to formulate the problem of finding redistricting plans as optimization problems on the basis of population equality and contiguity. The authors specifically address the problem from the contiguity perspective. They developed two exact optimal models: one based on a minimum spanning tree and one based on network flows. They discuss the spatial representation and the formulation of contiguity for both models and compare the performance of these two models, along with a third model developed in the literature, using a variety of synthetic and real data. The authors’ results confirm that such a problem is computationally intensive and more efficient methods are needed for large size problems, but with appropriate formulation approaches they can obtain useful baseline solutions to these problems with relatively small size. They also find that multiple optimal solutions with different spatial configurations may exist for the same problem, which presents a new challenge to the development of solution methods for political redistricting problems.

KEYWORDS

Land Acquisition Problems, Political Redistricting Problems, Spatial Contiguity, Spatial Optimization, Strict Equal Population

INTRODUCTION

Political redistricting is a process of redrawing the boundaries of political districts such that a set of criteria can be satisfied. The most prominent example is the redistricting of congressional districts in the United States where the districts are designed to fulfill the one-person-one-vote doctrine (Morrill 1981). Population equality is often required in this process as a rule that requires that all districts have approximately the same number of voters to overcome malapportionment. Contiguity, as required by most states as well as the supreme court rulings1, is a rule that states that an individual must be able to travel from any point in a district to any other point in that district without crossing the district’s boundary (Mills 1967; Nagel 1972; Grofman 1985).

Political redistricting has been and continues to be a highly controversial issue in the United States, partly because of a common practice that benefits a certain political group under the disguise

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of meeting other criteria, especially population equality. This process, known as Gerrymandering, normally lead to strange shapes of districts, which has made many researchers to believe that a computer created plan can be more “objective” (Altman 1997). Although computers have been used in redistricting since the 1960s (Thoreson and Liittschwager 1967), the computational intensity of computer programs has generally made them inferior to human experts (Altman 1995; Altman et al. 2005). The hope of using computers to automatically generate districting plans faded as researchers have realized that computer programs are no more objective than their human developers and users. Another reason is the complexity of the problem that has discouraged the use of computers in this area. The past decade has seen the development of many “perfect” redistricting plans where the population difference between any two districts is either zero or one person. These plans were created with geographical information systems that allowed interactive manipulation of spatial boundaries during the search for desirable plans. This situation presents a significant challenge: Is it possible to develop efficient algorithms or models that can compete with human?

Researchers have approached political redistricting problems as combinatorial optimization problems where the number of feasible solutions exponentially increases with the problem size (Altman 1998). These problems may be difficult to solve because of challenges in formulating their requirements in mathematical forms (Altman 1997; Altman 1998; Eagles et al. 2000), simultaneously satisfying several redistricting criteria (Williams 1995; Wei and Chai 2004), and the size of the solution space (Baço et al. 2005). A solution method for political redistricting problems should be efficient with respect to computational time and effective with respect to its ability to find high quality solutions.

In the literature of solving redistricting problems, one can notice that the majority of the solution methods are heuristic that cannot guarantee the optimal solution be found. This is mainly due to the difficulty in formulating spatial contiguity in the optimization model. Until relatively recently, the only exact method is the implicit enumeration method developed by Garfinkel and Nemhauser (1970). Starting in the late 1990s, new methods of modeling contiguity have emerged in different research areas. In the land acquisition problem literature, Cova and Church (2000) developed contiguity constraints based on the search for the shortest path between a land parcel and a pre-selected root land parcel. Williams (2002) developed a contiguity model based on the construction of a minimum spanning tree. Shirabe (2005) developed contiguity constraints based on the search for network flows. Many of these models have been applied in the development of exact models for political redistricting problems. For example, Shirabe (2009) proposed a contiguity-based exact optimization models to political redistricting based on land acquisition problems, and Duque et al. (2011) proposed developed three models based on the traveling salesman problem (Miller et al. 1960), geographical site design problem (Cova and Church 2000), and the flow network problem (Shirabe 2005).

The purpose of this paper is two fold. We discuss the development of two new models for political redistricting problems. The first model formulates spatial contiguity based on spanning tree (Williams 2002), and the second on the network flow model (Shirabe 2005). We must note here that the second model is similar to the flow model by Duque et al. (2011), which, as will be discussed below, is similar to the third model proposed by Shirabe (2009). The reason we still present the second model here is that our two models were developed approximately at the same as the other models (Reference will be included after the review process). Along with a long existing exact method, the implicit enumeration method by Garfinkel and Nemhauser (1970), we intent to fulfill a second goal of this paper by providing a systematic examination of the performance of these models. Much of the work in the literature chose test problems of less than 50 spatial units. We increase the problem size to a degree where all the models cannot return the optimization solutions in a relatively long time. Such a stress test can provide insight to the difficulty of the problem and will be useful in helping develop new methods.

Many other problems also share the same characteristics as the political redistricting. These problems can be found in applications such as sales districting (Hess and Samuels 1971; Zoltmers and Sinha 1983; Fleischmann and Paraschis 1988), school districting (Schoepfle and Church, 1989;
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