Chapter 3

He’s Variational Iteration Method

ABSTRACT

In this chapter, a variational iteration method (VIM) has been applied to nonlinear heat transfer equation. The concept of the variational iteration method is introduced briefly for applying this method for problem solving. The proposed iterative scheme finds the solution without any discretization, linearization, or restrictive assumptions. The results reveal that the VIM is very effective and convenient in predicting the solution of such problems.

INTRODUCTION

Basic Idea of He’s Variational Iteration Method

To illustrate the basic concepts of variational iteration method, we consider the following deferential equation:

\[ Lu + Nu = g(x), \]

where \( L \) is a linear operator, \( N \) a nonlinear operator, and \( g(x) \) a heterogeneous term. According to VIM, we can construct a correction functional as follows:

\[ u_{n+1}(x) = u_n(x) + \int_0^x \lambda \{ Lu_n(\tau) + Nu_n(\tau) - g(\tau) \} d\tau, \]

where \( \lambda \) is a general Lagrangian multiplier (He, 1998a, 1999b), which can be identified optimally via the variational theory (He, 1999b), the subscript \( n \) indicates the nth order approximation, \( \tilde{u}_n \) which is considered as a restricted variation, i.e. \( \delta \tilde{u}_n = 0 \).

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BACKGROUND

Most of engineering problems, especially some heat transfer equations are nonlinear, and in most cases it is difficult to solve them, especially analytically. Perturbation method is one of the well-known methods to solve nonlinear problems, it is based on the existence of small/large parameters, the so-called perturbation quantity (Cole, 1968; Nayfeh, 2000). Many nonlinear problems do not contain such kind of perturbation quantity, and we can use non perturbation methods, such as the artificial small parameter method (Lyapunov, 1992), the $\delta$ -expansion method (Karmishin, Zhukov & Kolosov, 1990), the Adomian’s decomposition method (Adomian, 1994), the homotopy perturbation method (HPM) (He, 2005, 2007; Ganji & Rajabi, 2006; He, 2006, 2000), and the variational iteration method (VIM) (He, 2000, 1998a; Ganji & Sadighi, 2006; Ganji, Jannatabadi & Mohseni, 2006).

Heat transfer equations in straight surfaces, are one most applicable of scientific problems. One of these surfaces is straight fins that are employed to enhance the heat transfer between the primary surface and its convective, radiating or convective-radiating environment. Extended surfaces are extensively used in various industrial applications (Kem & Kraus, 1972). Aziz and Hug (1975) used the regular perturbation method to obtain a closed form solution for a straight convecting fin with temperature dependent thermal conductivity. A method of temperature correlated profiles is used to obtain the solution of optimum convective fin when the thermal conductivity and heat transfer coefficient are functions of temperature (Sohrabpour & Razani, 1993).

Yu and Chen (1999) assumed that the linear variation of the thermal conductivity and exponential function with the distance of the heat transfer coefficient and then, solved the nonlinear conducting-convecting-radiating heat transfer equation by the differential transformation method.

MAIN FOCUS OF THE CHAPTER

In this chapter, the basic idea of VIM is introduced and then its application in some nonlinear heat transfer equations is studied. In addition, the nonlinear equations of the heat radiation and conduction equations of a fin in the steady state and in the free space are solved through VIM.

The VIM is strongly and simply capable of solving a large class of linear or nonlinear differential equations without the tangible restriction of sensitivity to the degree of the nonlinear term and also it reduces the size of calculations besides, its interactions are direct and straightforward.

In the last section, the mathematical model of variational iteration method is introduced and then its application in heat transfer equations is studied.

Nonlinear Equations Arising in Heat Transfer

Application

Example 1: Cooling of the lumped system with variable specific heat. Consider the cooling of a lumped system (Ganji, 2006), with volume, surface area $A$, density $\rho$, specific heat $C$ and initial temperature $T_i$. At time $t = 0$, the system is exposed to a convective environment at temperature $T_a$ with convective heat transfer coefficient $h$. Assume that the specific heat $C$, satisfies the following linear function temperature: