Function Projective Dual Synchronization with Uncertain Parameters of Hyperchaotic Systems

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ABSTRACT

Function projective dual synchronization between two pairs of hyperchaotic systems with fully unknown parameters for drive and response systems is investigated. On the basis of the Lyapunov stability theory, a suitable and effective adaptive control law and parameters update rule for unknown parameters are designed, such that function projective dual synchronization between the hyperchaotic Chen system and the hyperchaotic Lü system with unknown parameters is achieved. Theoretical analysis and numerical simulations are presented to demonstrate the validity and feasibility of the proposed method.

KEYWORDS
Adaptive Control, Dual Synchronization, Function Projective, Lyapunov Stability Theory, Unknown Parameters

1. INTRODUCTION

Chaotic systems are defined as nonlinear dynamical systems which are sensitive to initial conditions, topologically mixing and with dense periodic orbits (Azar et al., 2017a, 2017b; Ouannas et al., 2017a, 2017b, 2017c, 2017d, 2017e; Vaidyanathan et al., 2017a, 2017b, 2017c; Ouannas et al., 2016a, 2016b, 2016c, 2016d; Lazaros & Azar, 2017). Sensitivity to initial conditions of chaotic systems is popularly known as the butterfly effect. Small changes in an initial state will make a very large difference in the behavior of the system at future states. In the last five decades, there is significant interest in the literature in discovering new chaotic systems (Vaidyanathan & Azar, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f; Boulkroune et al., 2016a, 2016b). Synchronization of chaotic systems is a phenomenon that occurs when two or more chaotic systems are coupled or when a chaotic system drives another chaotic system (Vaidyanathan & Azar, 2015a, 2015b, 2015c, 2015d; Vaidyanathan et al., 2015a, 2015b, 2015c). In the last years, it has been shown that chaotic behavior is typical for three dimensional systems with only one positive Lyapunov exponent. In higher dimensional systems, it is possible to find hyperchaotic attractors with two (or more) positive Lyapunov exponents. These hyperchaotic systems have more complex behaviors and rich in dynamics compared to the chaotic systems. Laboratory experiments have also revealed the existence of hyperchaos in hydrodynamic systems and semiconductor device. The first hyperchaotic system was proposed by Rössler, (1979). Since regular chaotic systems have just one positive Lyapunov exponent, messages masked by regular chaotic systems are not always safe (Perez & Cerdeira, 1995). Therefore, it was suggested that this problem can be overcome by using higher-dimensional hyperchaotic systems, which have increased

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higher unpredictability. In view of the fact that a hyperchaotic attractor is characterized as a chaotic attractor with more than one Lyapunov exponent, dynamics of the system are expanded in more than one direction. Due to its higher unpredictability compared to regular chaotic systems, hyperchaos may be more useful in some fields such as communication, encryption, chemical reactions, electrical engineering, telecommunications, computing and information processing (Chen & Dong, 1998; Luo, 2009). Moreover, hyperchaos synchronization has attracted a great deal of attention from various scientific fields since Pecora and Carroll introduced a method to synchronize two identical chaotic systems with different initial. Many methods and techniques for handling hyperchaos synchronization have been developed, such as, complete synchronization (Carroll & Perora, 1990; El-Ghazal, 2012; Lin & Kuo, 2012), phase synchronization (Rosenblum et al., 1996), anti-phase synchronization (Al-sawalha 2013; Al-Sawalha & Noorani, 2008; Al-Sawalha & Noorani, 2010; Al-Sawalha & Noorani, 2009), lag synchronization (Chen et al., 2007; Shahverdiev et al., 2002), generalized synchronization (Rulkov et al., 1995), projective synchronization (Zhang et al., 2012; Chen et al., 2010; Sudheer & Sabir, 2010; Dua et al., 2010) and so on. Function projective synchronization, which is the generalization of projective synchronization, is one of the important synchronization methods that has been an active area of research. In function, projective synchronization the master and slave systems can be synchronized up to a desired scaling function (An & Chen, 2008; Chen & Li, 2007; Chen et al., 2008). Recently, the theory of dual synchronization in chaotic and hyperchaotic systems in which two different pairs of chaotic systems, i.e., two master systems and two slave systems are synchronized has been intensively reviewed and studied. The first study on dual synchronization of chaotic systems has been reported by Tsimring and Sushchik in 1996 in (Tsimring & Sushchik, 1996) Later, several dual synchronization methods have been reported. (Liu & Davids, 2000; Uchida et al., 2003; Ning et al., 2007; Hassan & Mohammad, 2008; Ghosh & Chowdhury, 2010; Ghosh 2011; Jian et al., 2013; Othman et al., 2016a; Othman et al., 2016b; Othman et al., 2016c; Othman et al., 2016d). On the other hand, at present, the function projective dual synchronization of hyperchaotic systems with unknown parameters have not yet been studied by any researcher. How to effectively function projective dual synchronize of a pair of a hyperchaotic systems with unknown parameters is an important problem for the theoretical and practical applications. As we know, we usually take adaptive control methods to deal with such problems. There are few results about dual synchronization of a pair hyperchaotic systems with fully unknown parameters not to mention function projective dual synchronization (Othman et al., 2016a; Othman et al., 2016b; Othman et al., 2016c; Othman et al., 2016d). Motivated by the above discussions, in this paper, we study function projective dual synchronization between hyperchaotic Chen system and hyperchaotic Li system with fully unknown parameters where only the master systems are assumed to be uncertainty and a novel parameter identification and an adaptive control law are derived based on the Lyapunov stability theory. Corresponding theoretical analysis and numerical simulations are carried out for function projective dual synchronization behavior of hyperchaotic systems with unknown parameters which are depicted through figures for different particular cases.

2. PROBLEM STATEMENT

Consider the following two hyperchaotic systems with uncertain parameters as the drive system:

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1) + F_1(x_1)\alpha, \\
\dot{y}_1 &= g_1(y_1) + G_1(y_1)\beta,
\end{align*}
\]

(1)

where, \( x_1 = \left( x_{11}, x_{12}, \ldots, x_{1n} \right)^T \in R^n \) and \( y_1 = \left( y_{11}, y_{12}, \ldots, y_{1n} \right)^T \in R^n \) are the state vectors of the systems, \( f_1 : R^n \rightarrow R^n \) and \( g_1 : R^n \rightarrow R^n \) are two continuous vector functions,
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