Chapter V

Malliavin Calculus for the Estimation of the U.S. Dollar/Euro Exchange Rate When the Volatility is Stochastic

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Abstract

The tendency of exchange rates to fluctuate markedly and regularly is often referred as currency market volatility. The extent of currency market volatility is a major element of market risk. For financial transactions, volatility represents both costs and profit opportunities. Increased currency market volatility implies higher currency option premia and, therefore, higher hedging costs for investors and importers/exporters. However, for banks and other investment houses dealing in options, an increase in option prices may contribute to higher profits. It has been well established that the volatility of exchange rates changes with time. In recent years, various stochastic volatility models have been proposed in the literature that try to capture the exchange-rate volatility dynamics. In turn, several methods have been developed to
estimate the parameters of such stochastic volatility models, with varying results. In this chapter, we propose another method for the estimation of the parameters of an exchange rate function when the volatility follows a stochastic process. Stochastic volatility is represented by a geometric Brownian motion. Using Malliavin calculus, we are able to find an explicit expression for the likelihood function of the observations. Numerical integration methods (Monte-Carlo simulations) and numerical optimization methods (generic algorithms) enable us to find an estimate for the unknown parameters and the volatility. This estimation method is then applied to the U.S. dollar/euro exchange rate. Specifically, first we formulate a U.S. dollar/euro exchange rate equation with a stochastic volatility model. We assume that the observed U.S. dollar/euro exchange rate follows a stochastic differential equation with random volatility, while the unobserved volatility follows a different stochastic differential equation. Then, we obtain the likelihood function of the observations by applying Malliavin calculus. The estimation of the unknown parameters is achieved through the maximization of the likelihood function. Using weekly U.S. dollar/euro exchange rates for the period April 28, 2000, to March 26, 2001, we obtain estimates of the parameters of the U.S. dollar/euro exchange rate function (i.e., the constant of the drift) and the assumed stochastic volatility model (i.e., the constants of the diffusion process). Application of the estimated model to out-of-sample data for the U.S. dollar/euro exchange rate shows a significantly high accuracy of the proposed method, as indicated by the very low root mean square error for the estimated exchange rate. This method can also be applied to other models of financial variables that follow similar processes.

Introduction

The magnitude of fluctuations among currencies is often called exchange rate volatility. It has been well established that currency market volatility could change rapidly over time. The extent of currency market volatility is a major element of market risk. For financial transactions, it represents both costs and profit opportunities. Currency market volatility raises the costs of hedging, for example, as indicated in the pricing of options. Increased volatility implies higher option premia and therefore higher hedging costs for investors and importers/exporters, but it may also contribute to generally higher profits for banks and other investment houses dealing in options (Papaioannou, 2001).

The observed instability in currency markets during the last two decades has been seen as a consequence of at least five identifiable factors: (i) The present floating exchange rate system, which allows for wide currency fluctuations; (ii) The increased global financial integration caused by the emergence of free trade blocks and new currencies such as the euro; (iii) The growth in capital flows as a result of the liberalization of trade in goods and services; (iv) The increased response of financial markets to emerging opportunities from interest rate differentials, misalignments and market inefficiencies; and (v) The spread of information technology.
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