Chapter 1

Basic Mathematical Structures

ABSTRACT

This chapter presents some conventional means of logical analysis. It is necessary to show the scope and features of our approach. In particular, we refer to algebra of sets and algebra of logic (propositional algebra), which belong to the class of Boolean algebras, as well as partially ordered sets and the theory of relations.

... If there are controversies among people, it would be sufficient just to say “Let us calculate!” in order to ... make clear who was right. – Gottfried Wilhelm Leibniz

INTRODUCTION

Here we review means of logical analysis, some of which have historically played an important role in shaping this scientific domain, while others are used in modern software systems in such areas as artificial intelligence and system analysis. These means can be divided into two categories: 1) systems based on the formal approach (i.e., TFS); 2) algebraic systems. Within TFS, in addition to classical logics (syllogistic, propositional logic, predicate logic, etc.), non-classical logics (default logic, non-monotonic logic, etc.) are actively developed. Non-classical logics are often used for modeling and analysis of defeasible reasoning (reasoning with hypotheses and abductive conclusions), but we support the opinion that it is advisable to solve this kind
of computer problems algebraically, by using laws and operations of algebra of sets. Subsequent chapters are devoted to a detailed presentation of this approach to logical analysis.

Among algebraic systems, providing solutions for a wide range of logical-analysis tasks, we consider algebra of sets and algebra of logic (propositional algebra), which belong to the class of Boolean algebras, as well as partially ordered sets and theory of relations.

**ALGEBRA OF SETS**

**Main Concepts**

Currently, algebra of sets is rarely used as an instrument of logical analysis. This is largely due to the undermining of confidence in the legitimacy of the set-based approach after the turn of the 19th and 20th centuries when paradoxes of set theory were formulated. The modern version of the set theory was created by Georg Cantor, who published 6 scientific memoirs on this subject from 1878 to 1884. These papers considered problems of equipotency for infinite sets, properties of totally ordered sets, measures for sets, etc. Later, Cantor, Burali-Forti C., and others discovered “paradoxical” sets. Their example, in particular, was formulated by Bertrand Russell in 1905 as “the set of all sets that does not contain itself as an element”. In more detail, this Russell’s paradox will be considered below.

Detection of such paradoxes led many logicians and mathematicians to believe that “naive” notion of the set is contradictory in principle. We will adhere to the point of view according to which the theory-of-sets paradoxes resulted more from logical inaccuracy in their formulation (non-noticeable to some people) rather than from objective reasons. A closer look reveals that most (and perhaps all) paradoxes arise from a thinly disguised play on words.

From the standpoint of modern mathematics, algebra of sets can be attributed to a class of algebraic systems usually defined as below.

An algebraic system is a structure containing the following components:

1. A carrier is a certain set of objects (e.g., numbers, geometric shapes, symbols, sets, etc.);
2. Totality of relations (such as “larger than”, “smaller than”, “equal to”, etc.);
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