Chapter 7
Stereohedrons and Partition of n–Dimensional Space

ABSTRACT

The process of hierarchical filling of space by p-dimensional regular polytopes is considered under the condition of large-scale discrete increase in the size of polytopes and preservation of their shape (scaling process). It is shown that the polytopic prismahedrons are a concrete realization of the stereohedrons. The polytopic prismahedrons have the necessary properties for translational filling of spaces of higher dimension without slits face to face. Moreover, it is proved that the polytopic prismahedrons forming such fillings can have common elements of any dimension included in the polytope. On the basis of the research carried out in spaces of higher dimension, a new paradigm for describing a discrete world has been put forward.

THE SCALING PROCESS AND HIERARCHICAL FILLING OF THE n-DIMENSIONAL SPACE

The problem of completing space by polyhedrons is one of the fundamental problems of mathematics, which has long attracted the attention of scientists. In 1900, D. Gilbert formulated 23 mathematical problems that require solution (Gilbert, 1901). One of these problems (eighteenth) was devoted to this question. It was formulated as follows: “Construction of space from congruent polyhedrons”. This problem is especially complicated in the case of n-dimensional spaces (Delone, 1969), and up to the end it has not been solved.

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to this day under these conditions. The discovery of the processes of scaling occurring in natural media substantially enriches the problem of filling spaces with polyhedrons. The idea of scaling was advanced by Kadanov in 1966 when analyzing the processes of a second-order phase transition (Kadanov, 1966). According to this idea, the elementary cells of the high-symmetry phase before the phase transition in the process combine with the formation of enlarged elementary cells of the low-symmetry phase. The existence of this process it was confirmed experimentally. The idea of scaling allowed Wilson (Wilson, 1971) and Fisher (Fisher, 1972) to describe the second-order phase transition by the system of Ginzburg-Landau differential equations and to investigate it (Zhizhin, 2014a, 2014b, 2014c). The idea of scaling was used in analyzing processes in other fields of physics, for example, in analyzing the growth (enlargement) of clusters (Krapivsky, Redner & Ben-Naim, 2010). In the diffraction patterns of quasi-crystals of various intermetallic compounds (Abe, Yan & Pennycook, 2004; Munkhopadhyay et al., 1993; Zhang & Kelton, 1993), one can also see the enlargement of the shape of the figures that unite the group of glowing points of the diffraction patterns (see Chapter 4, Figure 1). This enlargement includes an increasing number of these points. Moreover, this process has no limit both in the large and in the smaller side. A certain model of the process of enlargement of objects can be a similar increase in the object’s odds. This leads to representations about the hierarchical filling of space with some initial figure. The process of increasing the size of a figure is discrete and it is determined by the coefficient of geometric progression, the value of which depends on the shape of the figure. The concept of a growing geometric manifold was introduced and the hierarchical filling of the plane by various regular polygons (Zhizhin, 2010) was investigated, as well as the hierarchical filling of three-dimensional spaces by regular convex polyhedrons (Zhizhin, 2012; Zhizhin, 2014c).

Now, applying the methods developed in previous studies (Zhizhin, 2010, 2014c), we consider the hierarchical filling of a multidimensional space in the process of enlargement of multidimensional convex bodies. As was shown earlier when studying the hierarchical filling of a plane with polygons (Zhizhin, 2010), there can be various ways of filling it. In a multidimensional space are different ways of hierarchical filling with polytopes also. In Chapter 4, the method of hierarchical filling of space by polytopes was considered, based on the transition of a polytope to a dual polytope at each step of filling the space. In this case, investigating scaling processes, we need to consider a method for filling a space of higher dimension with polytopes, at which
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