Interval-Valued Complex Fuzzy Sets and Its Application to the Malaysian Economy

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ABSTRACT

In this article, the authors introduce two new operations for the interval-valued complex fuzzy set model, and study the fundamental algebraic properties of these new operations. The utility and applicability of the relations of this model is then demonstrated by applying it to an economics problem. The interpretation of this example is provided and supported by real-life incidences that took place in the Malaysian economy in recent years. Lastly, a brief comparative analysis of the IV-CFS model and other similar models are presented.

KEYWORDS
Complex Fuzzy Set, Decision Making, Fuzzy Set, Interval-Valued Complex Fuzzy Set

INTRODUCTION

Fuzzy sets (Zadeh, 1965) and intuitionistic fuzzy sets (IFSs) (Atanassov, 1986) are two of the most well-known and commonly used fuzzy models in literature. Many aspects of the fuzzy and IF sets such as the aggregation operators, logic theory, information measures and fuzzy goal programming have been extensively studied, and applied in decision making problems in different areas. These include among others, the areas of MCDM, pattern recognition, engineering and economics. Some of the recent works in these areas can be found in Semwal et al. (2015), Garg (2016a, 2016b, 2016c, 2016d), Singh & Garg (2016), Wan & Li (2014), Xu (2007, 2010) and Garg et al. (2017). We refer the readers to these works for further details pertaining to the fuzzy and IF models.

Although the fuzzy and IF sets are highly useful and applicable to many different areas, they have one major shortcoming: both lack the ability to model two-dimensional information that are synonymous with time-periodic phenomena. This led to the introduction of the complex fuzzy set (CFS) model (Ramot, 2002). The CFS model is characterized by two components, namely the amplitude term and the phase term, both of which make up the membership function of this model. The amplitude term represents the degree of belongingness of the elements with respect to the set of attributes that is being considered, whereas the phase term represents the periodicity of the elements. This phase term is the key defining feature of the CFS model that sets it apart from all other similar models in literature. Although this is a rather unexplored area of research, the CFS model has nevertheless been generalized to develop several improved hybrid models, the latest among which

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include the complex intuitionistic fuzzy soft set (CIFSS) by Kumar and Bajaj (2014) and the complex vague soft set (CVSS) and its relations by Selvachandran et al. (2016a, 2016b).

Although the CFS model is definitely superior to ordinary fuzzy sets and its hybrid models, it has one major shortcoming: its non-flexible membership structure that forces users to assign a single-value for the amplitude and phase terms of the membership function, even when dealing with situations that involve uncertain and subjective information. To address this problem, Greenfield et al. (2016) proposed the interval-valued complex fuzzy set (IV-CFS) model and went on to use this model to develop interval-valued complex fuzzy logic. This model provides an interval-based membership structure that allows users to record their hesitancy in assigning membership values. This feature and its ability to represent two-dimensional information makes it ideal to be used to handle uncertain and subjective information that are prevalent in most time-periodic phenomena in the real world. These reasons served as the motivation to choose the IV-CFS model and use it in modelling time-periodic phenomena in economics. Two new advanced operations are introduced for this model, and the algebraic properties of these operations are presented and verified. A brief comparative analysis of the IV-CFS model and similar models is provided in the last section.

**PRELIMINARIES**

The CFS model is an extension to fuzzy sets given in Definition 2.1, in which the range of the amplitude and phase terms extend over a unit circle in the complex plane. Some important concepts pertaining to CFSs are presented below, all of which are due to Ramot et al. (2002).

**Definition 2.1:** (Zadeh, 1965) A fuzzy set $A$ in a universe of discourse $U$ is characterized by a membership function $\mu_A(x)$ that takes values in the interval $[0, 1]$, where $A \subset U$ and $\mu_A(x) : x \rightarrow [0,1]$, for all $x \in U$.

**Definition 2.2:** A complex fuzzy set $A$ defined on a universe of discourse $U$ is characterized by a membership function $\mu_A(x)$ that assigns a complex-valued grade of membership in $A$ to any element $x \in U$. By definition, all values of $\mu_A(x)$ lie within the unit circle in the complex plane and are expressed by $\mu_A(x) = r_A(x)e^{i \omega_A(x)}$, where $i = \sqrt{-1}$, $r_A(x)$ and $\omega_A(x)$ are both real-valued, $r_A(x) \in [0,1]$ and $\omega_A(x) \in (0,2\pi)$. A complex fuzzy set $A$ is of the form:

$$A = \{(x, \mu_A(x)) : x \in U\} = \{(x, r_A(x)e^{i \omega_A(x)}) : x \in U\} \ (1)$$

**Definition 2.3:** Let $A$ and $B$ be two complex fuzzy sets on $U$ with membership functions $\mu_A(x) = r_A(x)e^{i \omega_A(x)}$ and $\mu_B(x) = r_B(x)e^{i \omega_B(x)}$, respectively. The set theoretic operations on $A$ and $B$ are given as follows:

The complement of $A$ denoted by $\overline{A}$, is defined as:

$$\overline{A} = \{(x, \mu_A(x)) : x \in U\} = \{(x, r_A(x)e^{i \omega_A(x)}) : x \in U\} \ (2)$$

where $r_A(x) = 1 - r_A(x)$, and $\omega_A(x) = 2\pi - \omega_A(x)$.

The union of $A$ and $B$, denoted by $A \cup B$, is defined as:
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