Existence of Fuzzy Zhou Bargaining Sets in TU Fuzzy Games

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ABSTRACT
This article contends that cooperative games have been studied extensively in the literature. A central question in cooperative games is to study solution concepts and their relationships, those well-known solution concepts include cores, stable sets, Shapley values, bargaining sets, and so on. In 1981, Aubin introduced cooperative fuzzy games with fuzzy coalitions which reflect the situations where agents have the possibility to cooperate with different participation level with values between $0$ and $1$, varying from non-cooperation (participation level $0$) to full cooperation (participation level $1$). Since then, cooperative fuzzy games have been studied extensively in the literature. In this article, the authors extend the concept of Zhou bargaining sets - one of three major bargaining sets in cooperative games - to its fuzzy version and prove the existence of fuzzy Zhou bargaining sets which extends the existence theorem for Zhou bargaining sets by Zhou.

KEYWORDS
Balanced Collections, Bargaining Sets, Fuzzy Zhou Bargaining Sets, Transferable Utility (TU) Games

1. INTRODUCTION
Let \( N = \{1, 2, \ldots, n\} \) be the set of \( n \) players. Any subset of \( N \) is called a coalition.

We first recall some basic concepts given in Zhou (1994). For a coalition \( S \) and a vector \( x \in \mathbb{R}^n \), we denote \( x(S) = \sum_{j \in S} x_j \).

1.1. Definition 1.1
An \( n \)-person game \( V \) is with transferable utility (a TU game or cooperative game in coalition form) if there exists a function \( v : \mathcal{P}^N \rightarrow \mathbb{R} \) such that for every \( S \subseteq N \), \( V(S) = \{ x \in \mathbb{R}^n \mid \sum_{j \in S} x_j \leq v(S) \} \).

Thus, every TU game \( V \) has an underlying function \( v \).

Cooperative games have been studied extensively in the literature. A central question in cooperative games is to study solution concepts and their relationships, those well-known solution concepts include core, stable set, Shapley value, bargaining set, and so on. There are three major types of bargaining sets: the first one, called classical bargaining set and denoted by \( M_i^{(1)} \), is defined by R. J. Aumann and M. Maschler in 1964; the second one, denoted by MB, is given by Mas-Colell

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in 1989; and the third one is given by Zhou in 1994. The existence theorem for the Aumann-Maschler bargaining set has been established independently by Davis and Maschler (1963) and Peleg (1963), the existence for Mas-Colell bargaining set in a weakly superadditive TU game has been provided by Vohra (1991), and the existence of Zhou bargaining set for a TU game is given by Zhou (1994).

Our focus in this paper is on fuzzy Zhou bargaining sets. Let us first recall the concept of Zhou bargaining set. A payoff configuration of a TU game $V$ is a pair $\{x, L\}$, where $x$ is a vector in $R^n$, $L = \{S_1, S_2, \ldots, S_r\}$ is a partition of $N$, and $x \in V(S_i)$ for each $S_i \in L$. Then the set of all payoff configurations is:

$$C = \bigcup_{L \in P} \left( \bigcap_{S_i \in L} V(S_i) \right)$$  \hspace{1cm} (1.1)

where $P$ is the set of all partitions of $N$.

Clearly, $C$ is nonempty, closed, comprehensive, bounded from above, and contains the origin (i.e., zero vector) as an interior point.

We now define Zhou bargaining set which is based on $C$. Let $V$ be a TU game and let $x \in C$ be a payoff vector. An objection at $x$ is a pair $\{S, y\}$, where $S$ is a non-empty coalition and $y$ is a vector with indices in $S$ satisfying $y \in V(S)$ and $y_i > x_i$ for each $i \in S$. A $Z$-counterobjection to this objection is a pair $\{T, z\}$, where $T$ is a coalition with $z \in V(T)$ satisfying that:

(A1) $T \setminus S \neq \emptyset$, $S \setminus T \neq \emptyset$, and $S \cap T \neq \emptyset$

(A2) $z_i \geq y_i$ for all $i \in S \cap T$, and $z_i \geq x_i$ for all $i \in T \setminus S$

A payoff vector $x \in C$ is said to belong to the Zhou bargaining set $ZB(V)$ if for any objection at $x$, there exists a $Z$-counterobjection to it.

According to Zhou (1994), one good part by defining bargaining set on $C$ is that it is free of any particular coalition structure. For the existence of Zhou bargaining sets, Zhou (1994) proved the following theorem.

**1.2. Theorem 1.2 (Zhou, 1994)**

The Zhou bargaining set $ZB(V)$ is nonempty for every TU game $V$.

In 1981, Aubin introduced cooperative fuzzy games with fuzzy coalitions which reflect the situations where agents have the possibility to cooperate with different participation level with values between 0 and 1), varying from non-cooperation (participation level 0) to full cooperation (participation level 1). Since then, cooperative fuzzy games have been studied extensively in the literature. Recently, Liu (2017) provided a close connection between fuzzy (Aubin) Mas-Colell bargaining sets and competitive equilibria in coalition production economies. In this paper, we will study fuzzy Zhou bargaining sets extend Theorem 1.3 to its fuzzy version in TU fuzzy games.

**2. FUZZY ZHOU BARGAINING SETS**

A fuzzy coalition is a vector $s \in [0, 1]^n$, namely, $s = (s_1, s_2, \ldots, s_n)$ with $0 \leq s_i \leq 1$ for each $i \leq n$. The $i$-th coordinate $s_i$ of $s$ is the participation level of player $i$ in the fuzzy coalition $s$. We use $F^N$ for the set of all non-zero fuzzy coalitions on player set $N$. Clearly, $F^N$ is an infinite set. For each $s \in F^N$, we define the carrier of $s$ by $\text{car}(s) = \{i \in N \mid s_i > 0\}$.
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