Percentile Matching Estimation of Zigzag Uncertainty Distribution

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ABSTRACT

The problem of estimating parameters involved in zigzag uncertainty distribution is considered in this article. Sensing the difficulties involved in the direct application of statistical estimation techniques for uncertainty distribution, the present article considers the application of the method of percentile matching for estimating the unknown parameters of zigzag uncertainty distribution. This article clearly establishes the fact that the percentile matching method gives better estimates when compared to the method of moments if sample percentiles of appropriate orders are used in the estimation process. Detailed numerical studies have been carried out using simulated datasets possessing different characteristics for identifying optimal orders of percentiles which give better estimates of parameters.

KEYWORDS
Method of Moments, Method of Percentile Matching, Zigzag Uncertainty Distribution

1. INTRODUCTION

Different types of uncertainties arise in real life situations. According to Liu (2008) randomness and impreciseness (fuzziness) are basic types of objective uncertainty and subjective uncertainty, respectively. Probability theory has been developed to handle random phenomena in which the events are well defined and considered not to have vagueness or uncertainty. The idea of fuzzy set theory has been introduced by Zadeh (1965) in order to deal with fuzziness through membership values. Later the concept of fuzzy graphs has been introduced by Rosenfield (1975). Fuzzy robust graph coloring problem has been discussed in Dey, Pradhan, Pal and Pal (2015). Vertex coloring of a fuzzy graph using alpha cut can be seen in Dey and Pal (2012). Interval type 2 fuzzy set in fuzzy shortest path problem is available in Dey, Pal and Pal (2016). Several works on the applications of fuzzy set theory have been carried out in different branches of statistics.

Liu (2007) introduced the concept of Uncertainty theory. According to Liu (2017), the concept of uncertainty theory is one of the options available to deal with indeterminate phenomena whose outcomes cannot be predicted in advance. It turned out to be the solution for problems in the contexts where no samples are available which creates difficulty in using probability theory for dealing with such situations. In such cases, opinions of the domain experts become the only choice for further study. Liu (2017) framed uncertainty theory to model the belief degrees of domain experts in various contexts. Belief degree refers to the belief levels of experts regarding the occurrence of particular events. Structural characteristics of uncertain measure have been discussed in Zhang (2011).
can find similarities in the process of developing uncertainty theory with that of probability theory. However, the ideas developed in uncertainty theory find applications in dealing with problems arising out of impreciseness created in non-stochastic manner.

Liu (2017) pioneered the uncertainty theory over the years and has developed several study areas similar to that existing in probability theory. Uncertain Measure, Uncertain Variable, Uncertain Programming, Uncertain Risk Analysis, Uncertain Reliability Analysis, Uncertain Propositional Logic, Uncertain Set, Uncertain Logic, Uncertain Inference, Uncertain Process, Uncertain Calculus, Uncertain Differential Equation, Uncertain Finance and Uncertain Statistics are some of the concepts developed under uncertainty theory. Liu (2017) gives a detailed explanation of these concepts. Studies on testing uncertain hypotheses about uncertainty distribution functions have been made by Wang, Gao and Guo (2012) and Sampath and Ramya (2013). The concept of uncertain random variables has been introduced by Liu (2013) as a mixture of uncertainty and randomness.

The estimation of parameters in uncertainty distributions has received the attention of researchers. Liu (2017) considered the least square estimates of parameters in linear uncertainty distribution for a given expert’s experimental data found with the help of MATLAB toolbox. Method of least squares, Method of moments and Delphi method are some of the existing methods of estimation available in the uncertainty literature. Wang and Peng (2014) explains the method of moments and gives an expression for finding the empirical moments of uncertain variables. Sheng and Kar (2015) derived the first two theoretical moments of a linear uncertain variable using the idea of inverse uncertainty distribution and also explains the central moments of an uncertain variable. One of the important methods for estimation available in the theory of estimation namely, method of maximum likelihood cannot be used due to the absence of the concept of density functions in uncertainty theory. Hence, alternative approaches for estimation of parameters involved in uncertainty distributions become necessary. Motivated by the method of percentile matching available in statistical theory of estimation, recently, Sampath and Anjana (2016) introduced the method of percentile matching for estimating the unknown parameters in Linear uncertainty distribution. The details of method of percentile matching used in statistical theory of estimation can be seen in Klugman, Panjer and Wilmot (2008). Following the approach pursued in the work related to the estimation of unknown parameters in linear uncertainty distribution, this article is devoted for studying the problem related to the estimation of unknown parameters involved in Zigzag uncertainty distribution. This article is organized as follows. Second section gives a detailed explanation on various basic concepts in uncertainty theory. Third section explains the method of percentile matching in detail. Fourth section describes different methods of estimation available in uncertainty theory. Detailed description of the experimental study carried out for the estimation of unknown parameters in zigzag uncertainty distribution is given in the fifth section. Findings and conclusions are presented in the sixth section.

2. UNCERTAINTY THEORY

This section discusses about various basic concepts in uncertainty theory with reference to Liu (2017).

Consider \( \Gamma \) as a nonempty set and \( \mathcal{L} \) be a \( \sigma \) -algebra over \( \Gamma \). Each element \( \Lambda \in \mathcal{L} \) is called an event. A number \( \mathcal{M}(\Lambda) \) indicates the level of occurrence of the event \( \Lambda \).

2.1. Uncertain Measure

A set function \( \mathcal{M} \) is said to be an uncertain measure if it satisfies the following three axioms:

**Axiom 1:** (Normality Axiom) \( \mathcal{M}(\Gamma) = 1 \).

**Axiom 2:** (Duality Axiom) \( \mathcal{M}(\Lambda) + \mathcal{M}(\Lambda^c) = 1 \).

**Axiom 3:** (Subadditivity Axiom) For every countable sequence of events:
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