Chapter 8
Introduction: MHR Method

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ABSTRACT

Computer vision needs suitable methods of shape representation and contour reconstruction. One of them, invented by the author and called method of Hurwitz-Radon Matrices (MHR), can be used in representation and reconstruction of shapes of the objects in the plane. Proposed method is based on a family of Hurwitz-Radon (HR) matrices. The matrices are skew-symmetric and possess columns composed of orthogonal vectors. 2D shape is represented by the set of successive nodes. It is shown how to create the orthogonal and discrete OHR operator and how to use it in a process of shape representation and reconstruction. Contour of the object, represented by successive contour points, consists of information which allows us to describe many important features of the object as shape coefficients. 2D curve modeling is a basic subject in many branches of industry and computer science.

INTRODUCTION

Method of Hurwitz-Radon Matrices (MHR), invented by the author, can be applied in reconstruction and interpolation of curves in the plane. The method is based on a family of Hurwitz-Radon (HR) matrices. The matrices are skew-symmetric and possess columns composed of orthogonal vectors. The operator of Hurwitz-Radon (OHR), built from these matrices, is described. Author explains how to create the orthogonal and discrete OHR and how to use it in a process of curve interpolation and two-dimensional data modeling. Proposed method needs suitable choice of nodes, i.e. points of the 2D curve to be interpolated or extrapolated: nodes should be settled at each extremum (minimum or maximum) of one coordinate and at least one point between two successive local extrema, and nodes should be monotonic in one of coordinates (for example equidistance). Created from the family of \( N-1 \) HR matrices and completed with the identical matrix, system of matrices is orthogonal only for vector spaces of dimensions \( N = 1, 2, 4 \) or \( 8 \). Orthogonality of columns and rows is very important and significant for stability and high precision of calculations. MHR method is modeling the curve point by point without using any formula.

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of function. Main features of MHR method are: accuracy of curve reconstruction depending on number of nodes and method of choosing nodes, interpolation of \( L \) points of the curve is connected with the computational cost of rank \( O(L) \), MHR interpolation is not a linear interpolation (Ullman & Basri, 1991). The problem of curve length estimation is also considered. Algorithm of MHR method and the examples of data extrapolation are described. Value anticipation is the crucial feature in risk analyzing and decision making. Risk analysis needs suitable methods of data extrapolation and decision making. Proposed method of Hurwitz-Radon Matrices (MHR) can be used in extrapolation and interpolation of curves in the plane. For example quotations from the Stock Exchange, the market prices or rate of a currency form a curve. This chapter contains the way of data anticipation and extrapolation via MHR method and decision making: to buy or not, to sell or not. Proposed method is based on a family of Hurwitz-Radon (HR) matrices. The matrices are skew-symmetric and possess columns composed of orthogonal vectors. The operator of Hurwitz-Radon (OHR), built from these matrices, is described. Two-dimensional data are represented by the set of curve points. It is shown how to create the orthogonal and discrete OHR and how to use it in a process of data foreseeing and extrapolation. MHR method is interpolating and extrapolating the curve point by point without using any formula or function.

Computer vision needs suitable methods of shape representation and contour reconstruction. One of them, invented by the author and called method of Hurwitz-Radon Matrices (MHR), can be used in representation and reconstruction of shapes of the objects in the plane. Proposed method is based on a family of Hurwitz-Radon (HR) matrices. The matrices are skew-symmetric and possess columns composed of orthogonal vectors. 2D shape is represented by the set of successive nodes. It is shown how to create the orthogonal and discrete OHR operator and how to use it in a process of shape representation and reconstruction. Then MHR method is generalized to Probabilistic Nodes Combination (PNC) method. This work clarifies the significance and novelty of the proposed method compared to existing methods (for example polynomial interpolations and Bézier curves). Previous published papers of the author were dealing with the method of Hurwitz-Radon Matrices (MHR method). Novelty of this monograph and proposed method consists in the fact that calculations are free from the family of Hurwitz-Radon Matrices. Problem statement of this monograph is: how to reconstruct (interpolate) missing points of 2D curve having the set of interpolation nodes (key points) and using the information about probabilistic distribution of unknown points. For example the simplest basic distribution leads to the easiest interpolation – linear interpolation. Apart from probability distribution, additionally there is the second factor of proposed interpolation method: nodes combination. The simplest nodes combination is zero. Thus proposed curve modeling is based on two agents: probability distribution and nodes combination. Significance of this book consists in generalization for MHR method: the computations are done without matrices in curve fitting and shape modeling, with clear point interpolation formula based on probability distribution function (continuous or discrete) and nodes combination. This book also consists of generalization for linear interpolation with different (no basic) probability distribution functions and nodes combinations. So this book answers the question: “Why and when should we use PNC method?” Curve interpolation represents one of the most important problems in mathematics and computer science: how to model the curve via discrete set of two-dimensional points? Also the matter of shape representation (as closed curve-contour) and curve parameterization is still opened. For example pattern recognition, signature verification or handwriting identification problems are based on curve modeling via the choice of key points. So interpolation is not only a pure mathematical problem but important task in computer vision and artificial intelligence. The monograph wants to approach a problem of curve modeling by