Innovative Genetic Algorithmic Approach to Select Potential Patches Enclosing Real and Complex Zeros of Nonlinear Equation

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ABSTRACT

In this article, an innovative Genetic Algorithm is proposed to find potential patches enclosing roots of real valued function \( f: \mathbb{R} \rightarrow \mathbb{R} \). As roots of \( f \) can be real as well as complex, the function is reframed on to complex plane by writing it as \( f(z) \). Thus, the problem now is transformed to finding potential patches (rectangles in \( \mathbb{C} \)) enclosing \( z \) such that \( f(z)=0 \), which is resolved into two components as real and imaginary parts. The proposed GA generates two random populations of real numbers for the real and imaginary parts in the given regions of interest and no other initial guesses are needed. This is the prominent advantage of the method in contrast to various other methods. Additionally, the proposed ‘Refinement technique’ aids in the exhaustive coverage of potential patches enclosing roots and reinforces the selected potential rectangles to be narrow, resulting in significant search space reduction. The method works efficiently even when the roots are closely packed. A set of benchmark functions are presented and the results show the effectiveness and robustness of the new method.

KEYWORDS
Complex Root, Genetic Algorithms, Potential Rectangles, Real Root, Search Space Reduction

1. INTRODUCTION

Most of the engineering problems are inherently nonlinear in nature leading to the problem of obtaining all roots of single nonlinear equations. Some such engineering problems are those concerned with stability of systems, vibrations, periodic processes, electrical circuits with alternating current sources, wave phenomena and so on. Solving such nonlinear equations involving transcendental terms is an ever-challenging problem to those seeking to obtain closely packed complex roots. The advent of high-speed digital computers gave us the opportunity to expand the search to find all roots (real and complex) of single nonlinear equations, from which one can select more useful roots. Furthermore, even a system of nonlinear equations can be reduced to a single equation, for which all roots can then be found.

The conventional numerical iterative methods to solve a nonlinear equation can be mainly categorized as open methods and interval based methods. Most of the open methods such as Newton-
Raphson method are point based approaches with one or more initial guess points and they generally converge fast as described (Press, 2007; Goyal, 2007). But, the main drawback here is about having a proper initial guess for roots as well as number of roots (Burden & Faires, 2001; Traub, 1964). Interval based methods such as Bissection method. Regula-falsi method are robust but they converge slowly (Ortega et al., 2014). Both the above methods deliver only one root at one execution of the algorithm. Furthermore, these methods do not guarantee to find exhaustive roots in the region of interest (ROI). Additionally, to find a complex root, it is generally required that the initial guess must be complex (Antia, 2012; Krantz, 1999).

The Newton and Halley Method for Complex Roots was presented by Yau and Ben-Israel (1998). Pan and Zheng (2011) proposed a matrix method to find real and complex roots of polynomials by making use of approximating the roots as the eigen values of the companion or generalized companion matrix associated with an input polynomial. Oftadeh developed a cubically convergent iterative method for finding complex roots of nonlinear equations (Oftadeh et al., 2010). Gritton, proposed global homotopy continuation procedures for seeking all roots of a nonlinear equation by considering several nonlinear equations related to chemical engineering problems (Gritton, Seader et al., 2001). Method suggested by Pourrajabian et al. (2013) made use of Genetic algorithms for solving nonlinear algebraic equations. Ariyaratne et al. (2016) proposed a modified firefly algorithm to solve univariate nonlinear equations possessing complex roots. Ujevic (2007) suggested an iterative method for solving nonlinear equations. But, it is observed that most of these methods cannot effectively find the roots (especially complex roots), if they are closely packed.

The authors (Nadimpalli et al., 2014) proposed an interval based Novel GA method to select potential intervals enclosing roots of a given nonlinear equation in one variable, by making use of the effectiveness of interval based methods coupled with the power of Genetic Algorithms (GA). The collection of roots in the given ROI \([a, b]\) will form a partially ordered sequence between \(a\) and \(b\). One can find a partition of \([a, b]\) that includes all these points and also containing the patches enclosing the roots, which was the aim of work in Novel GA method (Nadimpalli et al., 2014). Such a partition, which is a feasible solution for the problem is considered as a chromosome of a GA. GA work with probabilistic transition rules to guide their search from a population of possible, feasible solutions (not on a single point) by making use of randomized operators such as crossover, mutation and selection (Deb, 2001; Goldberg, D.E., 1989; & Holland, 1992). The number of roots of a function is not known a priori and hence it poses challenges while defining the length of chromosome \((LC)\) in GA. Novel GA method (Nadimpalli et al., 2014) has been developed to address this challenge as well as finding patches by embedding apt learning mechanism. The fitness function for each chromosome in GA is defined as the total number of all potential intervals in \([a, b]\) satisfying the condition that the product of function values at the end points of each interval is less than zero. To recommend effective concise patches enclosing roots, ‘Refinement technique’ was further introduced that is capable of addressing nonlinear equations with closely packed real roots, (Floudas & Pardalos, 2008, p. 1725).

The present study is an extension of Novel GA method (Nadimpalli et al., 2014), in which we propose a methodology to select potential patches (rectangles) enclosing real and complex roots of a given nonlinear equation that works efficiently even in case of closely packed complex roots. The prominent advantage of the proposed method is that it is free from initial guesses and human intervention. In the proposed method, it is only required to specify the approximate ROI for real and imaginary parts. It is not required to have any knowledge about the location of the roots or their
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