Neutrosophic Soft $\Gamma$-Semiring and Its Ideals

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ABSTRACT

As a generalization of fuzzy sets, soft set and neutrosophic sets are applied to many branches of mathematics to overcome the complexities arising from uncertain data. Combining these notions, in this article, the author initiates the study of $\Gamma$-semirings, an extension of semirings, and its ideals by neutrosophic soft sets. After defining some necessary definitions, they investigate neutrosophic soft $\Gamma$-semiring, neutrosophic soft ideals, idealistic neutrosophic soft $\Gamma$-semiring and obtain some of its characterizations.

KEYWORDS

$\Gamma$-Semiring, Ideal, Idealistic, Neutrosophic, Soft

1. INTRODUCTION

The theory of fuzzy sets, proposed by Zadeh (1965), has provided a useful mathematical tool for describing the behavior of the systems that are too complex or illdefined to admit precise mathematical analysis by classical methods and tools. As a generalization of fuzzy sets, the intuitionistic fuzzy set was introduced by Atanassov in 1986, where besides the degree of membership of each element there was considered a degree of non-membership with (membership value + non-membership value) ≤ 1.

There are also several well-known theories, for instances, rough sets, vague sets, interval-valued sets etc. which can be considered as mathematical tools for dealing with uncertainties. But all these theories have their inherent difficulties. To overcome these difficulties, Molodtsov (1999) proposed a completely new approach, called the soft sets which can be seen as a new mathematical tool for dealing with uncertainties. In the soft set theory, the problem of setting the membership function does not arise which makes the theory easily applies to many different fields. At present, works on soft set theory are progressing rapidly. Maji et al. (2002) pointed out several directions for the applications of soft sets. They also studied several operations on the theory of soft sets. Aktas et al. (2007) studied the basic concepts of soft set theory and compared soft sets to fuzzy sets and rough sets providing some example to clarify the difficulties. Maji et al. (2001), Feng et al. (2010) also studied fuzzy soft sets based on fuzzy sets and soft sets. In 2010, Feng et al. introduced an adjustable approach to fuzzy soft set based decision making. Feng et al. (2008) defined soft semiring, soft ideals on soft semiring and idealistic soft semiring. Wu et al. (2013) also obtain some characterizations of soft hemirings.

In 1998, F. Smarandache introduced the concept of “neutrosophy” – (French neutre < Latin neuter, neutral, and Greek sophia, skill / wisdom) means knowledge of neutral thought. While “neutrosophic” (adjective), means having the nature of, or having the characteristic of Neutrosophy. So, besides the “truth” / “membership” and “falsehood” / “non-membership” components that both appear in fuzzy logic / set, the neutral / indeterminate / unknown part represents the main distinction between “fuzzy”

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and “intuitionistic fuzzy” logic / set. As a result neutrosophic set can distinguish between absolute membership and relative membership. Smarandache used this in nonstandard analysis such as result of sport games (wining / defeating / tie), votes, from yes / no / NA, from decision making and control theory etc. combining the non-standard analysis with a tri-component logic / set / probability theory and philosophy. After that so many researchers, for example Sweety and Arockiarani (2014), Aydoğdu (2015), Bhowmik and Pal (2010), Broumi et al., (2014), Majumder and Samanta (2014), Peng et al. (2016), Peng et al. (2014), Şahin et al. (2015), Salama and Alblow (2012), Wang et al. (2011), Ye (2016), and Zhang et al., (2014) enriched the theory of neutrosophic logic.

In 2013, motivated by the idea of neutrosophic set and combining it with the theory of soft sets, P. K. Maji introduced and studied ‘Neutrosophic soft sets’. We know that, Semiring (Golan, 1999; Hebish and Weinert, 1998) is a well-known universal algebra. It is a generalization of an associative ring \((R, +, \cdot)\). If \((R, +)\) becomes a semigroup instead of a group then \((R, +, \cdot)\) reduces to a semiring. Semiring has been found very useful for solving problems in different areas of applied mathematics and information sciences, since the structure of a semiring provides an algebraic framework for modeling and studying the key factors in these applied areas. As a generalization of semiring, \(\Gamma\)-semiring was first studied by M. K. Rao (1995) as a generalization of \(\Gamma\)-ring as well as of semiring. In \(\Gamma\)-semirings, the properties of their ideals and their generalizations play an important role in their structure theory and useful for many purposes.

So, as a continuation of ‘Neutrosophic soft sets’, our main aim of this paper is to introduce ‘Neutrosophic soft \(\Gamma\)-semirings’ and study some basic results of its ideals.

2. PRELIMINARIES

We now recall following definitions for subsequent use.

2.1. Definition (Rao, 1995)

Let \(S\) and \(\Gamma\) be two additive commutative semigroups with zero. Then \(S\) is called a \(\Gamma\)-semiring if there exists a mapping \(S \times \Gamma \times S \to S((a, \alpha, b) \to a\alpha b)\) satisfying the following conditions:

I. \((a + b)\alpha c = a\alpha c + b\alpha c\)

II. \(a\alpha (b + c) = a\alpha b + a\alpha c\)

III. \(a(\alpha + \beta)c = a\alpha b + a\beta b\)

IV. \(a\alpha (b\beta c) = (a\alpha b)\beta c\)

V. \(0_s a\alpha = 0_s = a\alpha 0_s\)

VI. \(0_s a = 0_s = b0_s, a\)

for all \(a, b, c \in S\) and for all \(\alpha, \beta \in \Gamma\).

For simplification we write 0 instead of \(0_s\) and \(0_\Gamma\).

2.2. Example

Let \(S\) be the set of all \(m \times n\) matrices over \(Z^-\) (the set of all non-positive integers) and \(\Gamma\) be the set of all \(n \times m\) matrices over \(Z^-\); then \(S\) forms a \(\Gamma\)-semiring with usual addition and multiplication of matrices.

Throughout this paper, unless otherwise mentioned \(S\) denotes a \(\Gamma\)-semiring.
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