Chapter 3

Optimality Principles for Fuzzy Dual Uncertain Systems

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ABSTRACT

This chapter considers the extension of the calculus of variations to the optimization of a class of fuzzy systems where the uncertainty of variables and parameters is represented by symmetrical triangular membership functions. The concept of fuzzy dual numbers is introduced, and the consideration of the necessary differentiability conditions for functions of dual variables leads to the definition of fuzzy dual functions. It is shown that when this formalism is adopted to represent performance indexes for uncertain optimization problems, the calculus of variations can be used to establish necessary optimality conditions as an extension to this case of the Euler-Lagrange equation. Then the chapter discusses the propagation of uncertainty when the fuzzy dual formalism is adopted for the state representation of a time continuous system. This leads to the formulation of a fuzzy dual optimization problem for which necessary optimality conditions, corresponding to an extension of Pontryagine’s optimality principle, are established.

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INTRODUCTION

One major challenge when leading with optimization of uncertain systems has been to take profit of previous theoretical results obtained in deterministic frameworks, mainly when these results have an analytical form. In this direction, few effective studies have been performed until today while in this chapter an original approach is developed to extend the main results of continuous systems optimization theory to a class of uncertain systems. More specifically, in this chapter is considered the extension of the calculus of variations to the optimization of a class of fuzzy systems where the uncertainty of variables and parameters is represented by symmetrical triangular membership functions. In many situations, the basic uncertainty representation with respect to variables and parameters is given by real intervals with a central value which is naturally expected to be more representative. This can result either from the fact that possible intervals are effectively the only information about these variables and parameters or from the fact that diverse levels of uncertainty are attached to them and their symmetrical triangular approximation is a way to have a common representation of uncertainty. It is also important to note that for optimization purpose, different total orders can be defined for this class of fuzzy numbers allowing the definite comparison of different performance levels expressed in this formalism. In this chapter, first a connection is established between dual numbers (introduced originally for the design and analysis of kinematics for mechanical systems) and symmetrical triangular fuzzy numbers. This leads to the concept of fuzzy dual numbers to which the main dual calculus operations can be applied, easing the manipulation of this class of fuzzy numbers. Then, the consideration of the necessary differentiability conditions for functions of dual variables leads to the definition of fuzzy dual functions. The way in which uncertainty effects are produced as well as other mathematical properties of this type of functions are discussed. At this point it is shown that when this formalism is adopted to represent performance indexes for uncertain optimization problems, the calculus of variations can be used to establish of an extension of the Euler-Lagrange equation to this case. These necessary conditions for optimality can be easily completed in the same formalism with local second order optimal conditions. Then is discussed the propagation of uncertainty when the fuzzy dual formalism is adopted for the state representation of a time continuous system. Linear and nonlinear dynamics involving either initial variable uncertainty or parameter uncertainty, are considered. This leads to the formulation of a so called fuzzy dual optimization problem for which necessary optimality conditions, corresponding to an extension of Pontryagin’s optimality principle, are established. The main concepts presented in this chapter will be illustrated through examples while the solution of different uncertain optimization problems will be discussed to display the effectiveness of the proposed approach.
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