Chapter XXII
Imprecise Functional Dependencies

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INTRODUCTION

Functional dependencies represent a fundamental concept in the design of a database since they are capable of capturing some semantics of the data strongly connected with the occurrence of redundancy in a database. The development of applications working on fuzzy and multimedia databases has required the extension of the functional dependency notion to these new types of data. Unfortunately, the concept of imprecise functional dependence or fuzzy functional dependence (IFD, for short) has not had a cogent and largely accepted definition yet. In fact, in attempt to capture different aspects of this notion of many proposals of IFD definition exist in literature, all having semantics and objectives somewhat unclear, especially with respect to the concern of redundancy (Bosc, et al., 1994, Cubero & Vila, 1994, Raju & Majumdar, 1988, Tyagi, et al., 2005, Wang, et al., 2000). Moreover, the debate on the definition of the concept of fuzzy functional dependency seems to be still in progress, as shown by the following question: “But the question remains: are these extended notions of functional dependency a natural generalization?” (Tyagi et al., 2005).

The first problem that arises in the definition of a “well-defined” IFD is to avoid subjectivity. To this aim, in what follows, we report some pertinent dissertations from the literature. In particular, there is a conspicuous literature facing the problem of the definition of scientific concepts, and two behaviors are possible when the need for a new
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definition arises. Firstly, according to the received view, the fact that scientists do not agree, shows that the definition has not been given yet up to now. In fact, for Carnap (1950), the procedure of explication is “the transformation of an inexact, prescientific concept, the explicandum, into a new exact concept, the explicatum.” (p. 3). Secondly, according to the analytic method view (Cellucci, 1998; Cellucci, 2000), a definition is always an ongoing process, as it is based on the formulation of a hypothesis. Within this philosophy, a definition of the concept of IFD has only to rely on the requirements of the specific problem that a person is facing.

A better understanding of the concept of IFD is possible only by looking at the ongoing process of its definition, through a critical and unifying view. Therefore, the goal of this proposal is to present an overview of imprecise functional dependencies and to provide a critical discussion of the dependencies presented for fuzzy and multimedia data.

BACKGROUND

In the sequel, we use the following notation: \( t, t_1, t_2, \) etc are tuples; \( t[X] \) are the values of the tuple \( t \) corresponding to the set of attributes \( X \) (these values are also called a \( X \)-representation); \( A_k[t] \) is the \( k \)-th attribute value of the tuple \( t \); \( A_k[t_i, t_j] \) stand for the couple \( (A_k[t_i], A_k[t_j]) \).

In traditional relational databases a functional dependency (FD, for short) is defined as a constraint between two sets of attributes from the database. Given two sets of attributes \( X \) and \( Y \), a functional dependency between them is denoted by \( X \rightarrow Y \). The constraint says that, for any two tuples \( t_1 \) and \( t_2 \) having \( t_1[X] = t_2[X] \), then \( t_1[Y] = t_2[Y] \). More precisely, given a table \( R \),

\[
X \rightarrow Y \iff \forall t_1, t_2 \in R(t_1[X] = t_2[X] \Rightarrow t_1[Y] = t_2[Y]).
\]

This concept cannot be immediately applied to multimedia databases or to fuzzy databases, because of the impossibility or inopportunity to apply the strict equality. For example a dependency between electrocardiography (ECG) and the hearth sound (HS) exists if we relax the equality in such a way that we can reasonably compare two electrocardiography images and two hearth sounds. In other words, two electrocardiography images are usually considered “equals” even though they do not coincide perfectly.

Imprecise functional dependencies (IFDs) are extensions of classical FDs and they serve just to capture the relations between data when equality is relaxed.

In general, an imprecise functional dependency between two sets of attributes exists if “similar \( X \)-representations are associated to similar \( Y \)-representations”. Naturally, this expression of the IFD concept is too vague, as it specifies neither the concept of similar representations, nor the nature of the association between representations.

In the sequel we specify the basic constituents of an IFD: the fuzzy implication, the similarity between attributes, and the resemblance between tuples. Their use in the definition of IFDs will be clear in the next section.

Fuzzy Logical Implication

The concept of fuzzy implication is an extension of the classical one. In fact, while classical implication can be seen as an operation taking couples of binary values to a binary value, fuzzy implication is an operation which take two real values from the interval \([0,1]\) as input and associates a value always in \([0,1]\):

\[
\Rightarrow: \{0,1\} \times \{0,1\} \rightarrow \{0,1\} \quad \text{(classical implication)}
\]

\[
\Rightarrow_f: [0,1] \times [0,1] \rightarrow [0,1] \quad \text{(fuzzy implication)}
\]