Chapter 19

Incorporating Representation-Based Instruction Into Mathematics Teaching: Engaging Middle Schoolers With Multiple Representations of Adding Fractions

Fadime Ulusoy
Kastamonu University, Turkey

Lutfi Incikabi
Kastamonu University, Turkey

ABSTRACT

Developing students’ abilities in using and selecting appropriate representations and making transformations between representations is an important issue in mathematics education. Among the mathematical concepts, fractions have always reported as a challenging issue for students in the middle schools for a long time. In this sense, researchers have suggested the use of different representations in learning and teaching of fractions in order to enhance students’ conceptual understandings of fractions. Based on this suggestion, we provide some activities for engaging middle school students with multiple representations of fractions by focusing on addition operation. These activities are focused on the need to identify appropriate instructional strategies to enhance students’ communication and critical thinking, thus improving their academic performance in mathematics.

INTRODUCTION

At first glance, mathematics can be seen as an abstract world involving concepts, algorithms and symbols without any connection with real world (e.g. Cramer, 2003). For this reason, researchers emphasize the necessity of teaching mathematics as a unified system of concepts and operations based on certain patterns and relationships that exist in the real world (Nair & Pool, 1991; Resnick & Ford, 1984). From
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this necessity, mathematics educators have paid attention to how to teach mathematics to the students effectively for a long time. In this sense, many researchers argue for the importance of using appropriate representations in teaching and learning of mathematics in terms of having complete understandings of mathematical concepts, expressing mathematical ideas and relationship between concepts (Dreyfus & Eisenberg, 1996; Duval, 2006; Goldin & Shteingold, 2001). Furthermore, it is emphasized that representing a mathematical concept in multiple ways and making flexible transitions within and between different modes of representations have critical importance in terms of reaching a perfect and complete internalization of a mathematical concept (Dufour-Janvier, Bednarz & Belanger, 1987; Kaput, Blanton & Moreno, 2008; Lesh, 1999; National Council of Teachers of Mathematics [NCTM], 2000). Hence, the use of representations has been a crucial topic in learning of mathematics over the past three decades in standards of school mathematics for developing students' abilities to use appropriate representations and to make correct and robust translations among them (Common Core State Standards Initiative [CCSSI], 2010; Ministry of National Education [MoNE, 2013]; National Council of Teachers of Mathematics [NCTM], 2000; Van de Walle, Karp & Bay-Williams, 2010). However, studies focusing on students' abilities in use of representations indicate that middle school students have inadequate knowledge and ability to select appropriate representations and to transform from one representation to the others (Gagatsis & Elia, 2004; Neria & Amit, 2004).

Students’ poor representation transformation skills adversely affect their mathematical learning and problem-solving performances (Gagatsis & Shiakalli, 2004; Lesh, Post & Behr 1987). In this sense, researchers have begun to incorporate multiple representation-based learning environments into middle school mathematics. They reached substantial empirical evidence for the positive effects of multiple representation-based instructions on students’ conceptual understanding (Çıkla, 2004; Schnitz & Bannert, 2003; Tunç-Pekkan, 2015) especially in selection and use of appropriate type of representations. In this sense, we think that it is important to understand the effects of multiple representation-based environments in mathematics classrooms. Hence, researchers who seek and design alternative pedagogical instructions can be more aware of what kind of learning occurred in representation transformation processes. We also think that researchers may have chance to support students’ learning via representations by choosing and utilizing an appropriate type of methods, manipulatives or activities.

In mathematics education context, worldwide research on students’ understanding about fractions indicates that learning fractions is a complex and difficult process. For instance, according to the US National Assessment of Educational Progress (NAEP, 2007) study, while 60% of fourth grade students could not determine whether 1/4 is greater than 1/5, half of eight grade students could not order three fractions from least to greatest correctly. Research on fractions has reported that there are some barriers in students’ reasoning of fractions such as an early emphasis on whole-numbers (e.g. Behr, Harel, Post & Lesh, 1994) and insufficient abilities to transform between multiple representations. At this point, researchers suggest the use of external representations in instruction of fractions (Dreher, Kuntze & Lerman, 2016; Tunç-Pekkan, 2015).

Elementary mathematics curricula throughout the world and in Turkey give a strong emphasis to crucial role of using multiple representations in mathematics teaching (MoNE, 2013; NCTM, 2000). For example, NCTM (2000) states the following: Instructional programs from prekindergarten through grade 12 should enable all students to— (i) create and use representations to organize, record, and communicate mathematical ideas; (ii) select, apply, and translate among mathematical representations to solve problems; and (iii) use representations to model and interpret physical, social, and mathematical phenomena (p.67). Similarly, Turkish middle school mathematics curriculum (MoNE, 2013) emphasizes