Chapter 2
Boundary Complexes and Interior Points of the Polytopes

ABSTRACT

This chapter describes how the structure of a polytope of dimension n consisting of points of the boundary complex including a set of faces from zero to n - 1 and a set of interior points that are not belonging to the boundary complex is considered. The value is equal to the number of elements of the boundary complex, which the given element belongs, having dimension one greater than the given element of the boundary complex is denoted coefficient incidence of the given element. It is proven that the coefficient incidence of an element of dimension i of the boundary complex of an n - cube and n - simplex is equal to the difference of the dimension of the cube or simplex n and the dimension of this element i. The incidence coefficient of elements of n – cross - polytopes is substantially higher than this difference.

INTRODUCTION

A finite family C of convex polytopes will be called a complex under the condition

1. Each face of an element of the family C is an element of the family C;
2. The intersection of any two elements of C is a face of both.

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Let \( P \) be an \( n \)-polytope, i.e. a convex polytope of dimension \( n \). We denote by \( B(P) \) the boundary complex \( P \), that is, a complex consisting of all faces of \( P \) with dimension \( n - 1 \) or less (Grunbaum, 1967). The points not belonging to \( B(P) \) will be called interior points of \( P \). The set of interior points of the polytope \( P \) will be denoted by \( V \). Thus, we have equality \( P = B(P) \cup V \).

Since all the faces of a polytope from vertices (which in chemical compounds are images of atoms, molecules or functional groups) to facets enter the boundary complex, it is of interest to establish relationships between faces of different dimensions in the boundary complex \( B(P) \).

For each element \( b_j \) of the boundary complex \( B(P) \) can introduce the notation its dimension \( i_{b_j} \), \( 0 \leq i_{b_j} < n \). There is denoted \( k_{b_j} \) the incidence value of the element of the boundary complex \( b_j \) to the elements of the boundary complex having dimension \( i_{b_j} + 1 \). This value is equal to the number of elements of the boundary complex which the given element belongs having dimension one greater than the given element of the boundary complex.

**THE STRUCTURE OF THE BOUNDARY COMPLEX OF A \( n \)-CUBE**

If polytope \( P \) is point (vertex), so evidence that \( n = 0 \), \( i_p = 0, k_p = 0 \). If polytope \( P \) is segment with vertices \( b_1, b_2 \) so the boundary complex \( B(P) \) is points \( b_1, b_2 \). In this case are \( n = 1, i_{b_1} = 0, i_{b_2} = 0, k_{b_1} = 1, k_{b_2} = 1 \). Can introduce the origin of coordinates \( O \) in the middle of the segment \( [b_1, b_2] \). Thus, if put of length of the segment equal one, so the boundary elements (points) have coordinates \( x_{b_1} = -\frac{1}{2}, x_{b_2} = \frac{1}{2} \). The set of interior points in this case is \(-\frac{1}{2} < x < \frac{1}{2}\). If \( n = 2 \) so polytope \( P \) is square (Figure 1). There elements of the boundary complex are vertices \( b_1, b_2, b_3, b_4 \) and edges \( b_1b_2, b_2b_3, b_3b_4, b_4b_1 \). The dimensions of the vertices equal zero \( i_{b_1} = 0, j = 1 \div 4 \). The dimensions of the edges equal one \( i_{b_1b_2} = i_{b_2b_3} = i_{b_3b_4} = i_{b_4b_1} = 1 \). The incidence value of the elements of the boundary complex equal corresponding
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