Solving the Maximum Clique Problem using a Hybrid Particle Swarm Optimization Algorithm

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ABSTRACT

This article tackles the maximum clique problem MCP known as an NP-hard graph problem. The maximum clique problem consists in finding in an undirected graph a complete sub-graph (clique) of maximum cardinality. As the MCP is a classical graph problem extensively studied, the main contribution of this paper is to use for the first time particle swarm to solve it. A hybrid particle swarm optimization algorithm HPSOD is proposed. First a PSO algorithm is designed, based on a sub-graph extraction approach named circular-arc graph CAG, then a local search heuristic is integrated to enhance its performance. Experimental tests carried out on DIMACS benchmarks show a globally good performance of the proposed algorithm and that it outperforms many existent approaches.

KEYWORDS

Circular-Arc Graph Algorithm, Hybrid Metaheuristic, Hybrid Particle Swarm Optimization, Local Search, Maximum Clique

INTRODUCTION

This paper deals with the maximum clique problem (MCP). In this problem, we are given an undirected graph G = (V, E), where V = {v1, ..., vn} is the set of n vertices and E ⊆ V x V, the set of m edges. The maximum clique problem MCP consists of finding a clique with the maximum cardinality. A clique C of an undirected graph is a complete sub-graph. A graph is said to be complete if all its vertices are pairwise adjacent. The maximum clique problem is an important combinatorial optimization problem with a variety of applications in different fields such as classification theory, bioinformatics, signal transmission, examination timetabling and scheduling. It’s also related to many important graph problems that can be either formulated as a clique problem or have sub-problems which can be solved using the maximum clique, like the graph coloring, optimal winner determination and sum coloring. The maximum clique has been proved to be NP-hard for arbitrary graphs (Garey & Johnson, 1979). Only particular classes of graphs are solved in polynomial time such as interval graphs, circular-arc graphs and triangulated graphs (Golumbic, 2004).

Given the theoretical and practical importance of the problem, it has been widely studied and several metaheuristic approaches have been proposed to solve it, for instance, genetic algorithms (Murthy, Parthasarathy, & Sastry, 1994), variable neighborhood method (Hansen, Mladenovic, & Urosevic, 2004), simulated annealing (Gibbons, Hearn, & Pardalos, 1996) and memetic algorithm (Dang & Moukrim, 2012). Metaheuristics are successful methods that have been commonly used to find near-optimal solutions in a reasonable computing time for many NP-hard combinatorial

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optimization problems and case studies (Samanta & Jha, 2012; Adamidis & Kynigopoulos, 2014; Sabba & Chikhi, 2015; Lienland & Zeng, 2015).

This work proposes to solve the maximum clique problem by a hybrid particle swarm optimization. To the best of our knowledge there is no particle swarm optimization (PSO) algorithm developed for this problem. Thus, the main objective of this paper is to investigate the capability of PSO in solving this classical graph problem. In this approach, initially a set of particles is created using a local search heuristic, where each particle corresponds to a possible solution (clique). For each particle, a permutation of vertices is generated by combining the current position, the best position and the best particle in the swarm. Then, the sub-graph extraction approach is applied to extract a new clique from the permutation. A local search is then integrated to enforce the effectiveness of the basic discrete particle swarm optimization PSOD algorithm.

The remainder of this paper is organized as follows: Section 2 presents the related works, and then a brief description of the classical Particle Swarm Optimization metaheuristic is given in section 3. In section 4, the proposed basic PSOD algorithm is described, followed by the hybrid algorithm HPSOD obtained by integration of local search. Finally, results of experimental tests carried out on DIMACS benchmark instances and comparison with the existing solving methods are provided in Section 5.

RELATED WORK

Several exact methods, heuristics and metaheuristics have been proposed to solve the maximum clique problem (MCP). For the exact methods, most of them are based on branch and bound framework, for example, an early basic branch and bound algorithm of Carraghan and Pardalos (1990). These methods differ according to their branching strategy or the technique of determination the bounds. Some exact methods are based on solving sub-clique problem such as the method of Ostergaard (2002), while others are based on sub-graph coloring like the algorithm of Babel and Tinhofer (1990), Tomita and Kameda (2007). Some are based on Maxsat such as the algorithm of Li and Quan (2010).


Clearly, the maximum clique is one of the most studied optimization problem and the number of works dealing with it is extensive. Thus, in this section only the main results are provided. For a detailed review of the existing methods, the reader is referred to a very recent survey of Wu and Hao (2015).

BASIC PRINCIPLES OF PARTICLE SWARM OPTIMIZATION METAHEURISTIC

Particle Swarm Optimization PSO is an evolutionary technique inspired from the behavior of wild animals in the nature. It was originally proposed by Kennedy and Eberhart (1995). PSO was first used for solving optimization problems in continuous space, and then it becomes useful for combinatorial optimization problems. A swarm is composed of candidate solutions called particles moving in the search space. Each particle has three possible movements: following its own way, going back to its local best previous position $X_{i}^{\text{Best}}$ according to an evaluation function called Fitness, and going towards the best position visited in the whole swarm known as global best $X_{g}^{\text{Best}}$. The swarm explores the search space according to equations (1) and (2)
An Integrated Vendor-Buyer Model with Uncertain Lead Time, Life Time under Inflation and Variable Holding Cost


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