Eigenvalue of Intuitionistic Fuzzy Matrices Over Distributive Lattice

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ABSTRACT

In this article, the concepts of intuitionistic fuzzy complete and complete distributive lattice are introduced and the relative pseudocomplement relation of intuitionistic fuzzy sets is defined. The concepts of intuitionistic fuzzy eigenvalue and eigenvector of an intuitionistic fuzzy matrixes are presented and proved that the set of intuitionistic fuzzy eigenvectors of a given intuitionistic fuzzy eigenvalue form an intuitionistic fuzzy subspace. Also, the authors obtain an intuitionistic fuzzy maximum matrix of a given intuitionistic fuzzy eigenvalue and eigenvector and give some properties of an intuitionistic fuzzy maximum matrix. Finally, the invariant of an intuitionistic fuzzy matrix over a distributive lattice is given with some properties.

KEYWORDS

Intuitionistic Fuzzy Eigenvalue, Intuitionistic Fuzzy Eigenvector, Intuitionistic Fuzzy Invariant Matrix, Intuitionistic Fuzzy Matrices

1. INTRODUCTION


The eigenvector-eigenvalues problem of matrices over distributive lattices have appeared firstly in the work of Rutherford (1965). He showed that given an eigenvector of a matrix over Boolean algebra the eigenvalues of the matrix with the given eigenvector vector form an interval in this algebra and give an explicit formula for this interval. Kirkland and Pullman (1992) presented a convenient way to construct spanning set of minimum cardinality for each eigenspace in the case of finite Boolean algebras. Sanches (1978) presented a way to find the maximum standard eigenvector of a given fuzzy matrix. Cechlarova (1992) characterized the standard eigenvectors by means of the associated graph of a matrix over the Bottleneck algebra. Gavalec (2002) studied the set of all increasing eigenvectors of a matrix over a max-min algebra.

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The fuzzy eigenvector and eigenvalue of a fuzzy matrix play a central role in mathematics and engineering. Fuzzy eigenvalues were first studied in Buckley (1990) to analyze input-output of systems. Massa (2008) determined both the fuzzy eigenvalues and eigenvectors of a finite model defined with fuzzy parameters. Alevizos et al. (2007) converted the fuzzy eigenvalue problem into an ordinary one in order to study correspondence data. To produce a fuzzy eigenvector weight estimate, Wang and Chin (2006) used the ordinary eigenvector through the solution of a linear programming model. Our aim in this study is to introduce the intuitionistic fuzzy eigenvalue and eigenvector for an intuitionistic fuzzy matrix (IFM) over the complete and complete intuitionistic fuzzy distributive lattice.

The structure of this paper is organized as follows. In Section 2, the preliminaries and some definitions are given. In Section 3, the concept of pseudocomplement relation is introduced. In Section 4, intuitionistic fuzzy eigenvalue and eigenvector are defined and the maximum intuitionistic fuzzy eigenvector of a given intuitionistic fuzzy matrix for a given intuitionistic fuzzy eigenvalue are obtained. Also, some properties of maximum intuitionistic fuzzy matrix from the set of intuitionistic fuzzy matrices with a given intuitionistic fuzzy eigenvalue and eigenvector are studied. In Section 5, invariant intuitionistic fuzzy matrix is defined and some properties of invariant intuitionistic fuzzy matrix are presented. Conclusions are made in Section 6.

2. PRELIMINARIES

In this section, we first give some preliminaries, definitions of IFSs and IFMs and then present some algebraic operations of IFMs and different types of IFMs (Adak and Bhowmik 2011, Adak et al. 2011a, b; Adak et al. 2012a, b, 2013).

2.1. Intuitionistic Fuzzy Set

**Definition 2.1** (Intuitionistic fuzzy set (IFS)) An intuitionistic fuzzy set (IFS) $A$ over $X$ is an object having the form $A = \{x, (\mu_A(x), \nu_A(x)) : x \in X\}$; where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, where $\mu_A(x)$ and $\nu_A(x)$ are the membership and non-membership values of $x$ in $A$ satisfying the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

**Lemma 1** (Poset of IFSs) Let $L$ be an IFS and ‘≤’ be comparable intuitionistic fuzzy relation, then $(L, \leq)$ is a poset.

**Proof.** Let $a, b, c \in L$ where $a = \langle a_\mu, a_\nu \rangle$, $b = \langle b_\mu, b_\nu \rangle$ and $c = \langle c_\mu, c_\nu \rangle$ then

1. $a \leq a$ is true since $a_\mu \leq a_\mu$ and $a_\nu \geq a_\nu$. Hence the relation ‘≤’ is reflexive.
2. $a \leq b$ and $b \leq a$ possible only when $a = b$, since $a \leq b$ when $a_\mu \leq b_\mu$ and $a_\nu \geq b_\nu$ and $b \leq a$ when $b_\mu \leq a_\mu$ and $b_\nu \geq a_\nu$. Combining these results give $a = b$. Therefore, the relation ‘≤’ is anti-symmetric.
3. $a \leq b$ and $b \leq c$ when $a_\mu \leq b_\mu$ and $a_\nu \geq b_\nu$ and $b \leq c$ when $b_\mu \leq a_\mu$ and $b_\nu \geq a_\nu$. Then it is obvious that $a \leq c$ since $a_\mu \leq c_\mu$ and $a_\nu \geq c_\nu$. Hence the relation ‘≤’ is transitive.

Therefore, $(L, \leq)$ is a poset.

For any two elements $a$ and $b$ in $L$, the meet and join will be denoted by $a \wedge b$ and $a \vee b$ and is defined by $a \wedge b = \langle a_\mu \wedge b_\mu, a_\nu \vee b_\nu \rangle$ and $a \vee b = \langle a_\mu \vee b_\mu, a_\nu \wedge b_\nu \rangle$ respectively.

2.2. Lattice of IFSs

A non-empty intuitionistic fuzzy poset $(L, \leq)$ with two binary operation $\vee$ and $\wedge$ is called a lattice if the following axioms hold:
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