On Multi-Fuzzy Rough Sets, Relations, and Topology

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ABSTRACT

This article describes how rough set theory has an innate topological structure characterized by the partitions. The approximation operators in rough set theory can be viewed as the topological operators namely interior and closure operators. Thus, topology plays a role in the theory of rough sets. This article makes an effort towards considering closed sets a primitive concept in defining multi-fuzzy topological spaces. It discusses the characterization of multi-fuzzy topology using closed multi-fuzzy sets. A set of axioms is proposed that characterizes the closure and interior of multi-fuzzy sets. It is proved that the set of all lower approximation of multi-fuzzy sets under a reflexive and transitive multi-fuzzy relation forms a multi-fuzzy topology.

KEYWORDS

Approximation Operators, Closed Multi-Fuzzy Sets, Closure, Interior, Multi-Fuzzy Relation, Multi-Fuzzy Rough Sets, Multi-Fuzzy Sets, Multi-Fuzzy Topology

1. INTRODUCTION

As per the traditional view, uncertainty is undesirable in science. But when science is used to solve the real-world problems, it is difficult to completely neglect their uncertainty side. Philosophers, computer scientists, logicians constantly confront situations of ambiguity and vagueness. The major turning point in the evolution of the modern concept of uncertainty was the publication of a paper on fuzzy sets by Lotfi A. Zadeh (1965). He introduced fuzzy sets as the sets with imprecise boundaries. Unlike classical set theory in which an element either belongs to a set or not, fuzzy set theory allows an element to belong to the set with a degree of membership. In the later years, fuzzy set theory attracted the interest of many researchers and various extensions and generalizations of fuzzy sets are proposed in the literature. L-fuzzy sets (Goguen, 1967), level-k fuzzy sets (Zadeh, 1975), type-2 fuzzy sets (Mizumoto & Tanaka, 1976), intuitionistic fuzzy sets (Atanassov, 1986), interval valued fuzzy sets (Gorzalczyz, 1987) are to name a few.

The theory of multi-fuzzy sets introduced by Sabu and Ramakrishnan (2011) is one such generalization. Multi-fuzzy sets are characterized by ordered sequences of membership functions. The theory helps to model many practical problems that demand multi-membership functions. The multi-fuzzy set theory provides a platform to deal with problems in the fields of image processing, taste recognition and pattern recognition. Decision making that involves more than one variable is made easy with the help of multi-fuzzy sets.

Multi-fuzzy rough sets are hybrid structures involving multi-fuzzy sets and rough sets. Researchers often combine several techniques to obtain a more superior technique which has the advantages of existing methods. The theory of multi-fuzzy rough sets makes an effort towards approximation of vague data in a multi-dimensional aspect and helps in handling data that includes numerous parameters.

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2. LITERATURE SURVEY

The chief aspect of fuzzy sets is that their boundaries are not precise. There exists an alternative way to formulate sets with imprecise boundaries. Sets formulated in this way are called rough sets. Zdzisław Pawlak (1982) laid the foundation of rough set theory. A rough set is basically an approximate representation of a given crisp set in terms of two subsets of a crisp partition defined on the universal set involved. The two subsets are called a lower approximation and an upper approximation. Rough set theory, has far reaching applications in numerous fields like data mining, machine learning, knowledge discovery, pattern recognition etc. The theory of rough sets and its various applications are well covered in (Pawlak, 2012) and (Polkowski, 2013).

Rough set theory and fuzzy set theory are not rivaling theories but rather complement each other. Both are different and independent approach to imperfect knowledge. Scholars like Dubois and Prade (1990) carried out studies on combining fuzzy set theory and rough set theory and obtained hybrid structures namely rough fuzzy sets and fuzzy rough sets. Guided by this observation, rough set theory is integrated with other fuzzy set theories so that more general structures like interval valued fuzzy rough sets (Zhang, Zhang, & Wu, 2009; Zhang, 2013) intuitionistic fuzzy rough sets (Zhou, Wu, & Zhang, 2009; Zhou & Wu, 2008), hesitant fuzzy rough sets (Deepak & John, 2014) multi-fuzzy rough sets (Varma & John, 2014) etc. are obtained. These investigations have shown how soft technology theories can be combined fruitfully to attain a more flexible and expressive framework that helps in modelling incomplete information in the available knowledge base.

Topology is one of the well-established and basic disciplines of mathematics. Many of the concepts of general topology is extended to fuzzy set theory. The idea of fuzzy topological space is initiated by Chang in (Chang, 1968). Later, Lowen (1976) suggested an alternative and more natural definition of fuzzy topology. Coker (1997) extended this concept to intuitionistic fuzzy sets. The notion of Multi-fuzzy topology in Chang’s sense is proposed in (Sebastian & Ramakrishnan, 2011a) . The concepts of continuity and compactness in multi-fuzzy sets are also studied. The relation between rough sets and topology has been a topic of research for a decade. The lower and upper approximation operators are the indispensable part of rough set theory. These operators can be seen as topological operators induced by the indiscernibility relation on the universe. The topological properties of rough sets need to be studied to improve the applications of topology and rough set theory on uncertain and incomplete information. The relationship between generalized rough sets and topologies is studied in (Lashin, Kozae, Abo Khadra, & Medhat, 2005; Pei, Pei, & Zheng, 2011; Wu, Wang, Huang, & Li, 2008; Kondo & Dudek, 2006). The topological structures of rough sets in fuzzy environment are discussed in (Qin & Pei, 2005; Srivastava & Tiwari, 2003) respectively. Wu & Zhou (2011) investigated the topological structures of intuitionistic fuzzy rough sets whereas Wang extended the studies to generalized fuzzy rough sets (Wang, 2017) and L-fuzzy rough sets (Wang, 2017). Covering approximation space and the topology induced on it is studied in (Zhao, 2016) and (D’eer and Cornelis, 2017).

In this paper, the authors look into the concept of multi-fuzzy topology through closed multi-fuzzy sets. It is seen that we could as well take ‘closed multi-fuzzy sets’ as a primitive concept and then define open multi-fuzzy sets as complements of closed multi-fuzzy sets. In section 2, the basic definitions and some known results regarding multi-fuzzy sets and multi-fuzzy rough sets are reviewed. The next section deals with the axiomatic characterization of closure and interior operator. The final section discusses the topological structure of multi-fuzzy rough sets that are induced by different types of multi-fuzzy relations.

3. BASIC CONCEPTS

In this section, the basic definitions and properties related to multi-fuzzy sets which will be used in this paper, are presented.
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