Chapter XLIV

Dengue Fever: A Mathematical Model with Immunization Program

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ABSTRACT

Dengue fever is a re-emergent disease affecting more than 100 countries. Its incidence rate has increased fourfold since 1970 with nearly half the world’s population now at risk. In the chapter, a mathematical model with immunization is proposed to simulate the succession of 2 epidemics with variable human populations. Stability of the equilibrium points is carried out and simulation is given for different parameters settings.

INTRODUCTION

At the dawn of the third millennium, the world population is facing a double burden of non communicable diseases (NCDs) and infectious diseases (Boutayeb, 2006). NCDs, once known as the disease of “the rich”, are now also affecting developing countries where Cardio-Vascular Diseases (CVDs), cancer and diabetes are flourishing (WHO, 2003; Boutayeb, 2005; Parkin, 1999). In parallel, infectious diseases continue to be the major causes of mortality and morbidity in low and middle income countries, where, well known existing, emerging and re-emerging diseases like tuberculosis, cholera, meningitis, hepatitis, malaria, dengue, yellow fever, AIDS, Ebola, SARS and others are causing suffering and mortality to a wide population. Among the infectious diseases, dengue fever, especially known in Southeast Asia, is now endemic in more than 100 countries world-wide. Its incidence has increased fourfold since 1970 and nearly half the world population (2.5-3 billion) is now at risk. It estimated that more than 50
million people are infected every year of which half a million of Dengue Haemorrhagic Fever (DHF) (DengueNet, 2007; Reprot, 2002; Teixeira, 2002). The two recognised species of the vector transmitting dengue are Aedes aegypti and Aedes albopictus. The first is highly anthropophilic, thriving in crowded cities and biting primarily during the day while the later is less anthropophilic and inhabits rural areas. Consequently, the importance of dengue is two-fold:

- With increasing urbanisation, crowded cities, poor sanitation and lack of hygiene, environmental conditions foster the spread of the disease which, even in the absence of fatal forms, breeds significant economic and social costs (absenteeism, immobilisation, debilitation, medication).
- The potential risk of evolution towards the haemorrhagic form and the dengue shock syndrome with high economic costs and which may lead to death.

Many authors have presented the disease as a major health problem either for the last decades of the 20th century or for the third millennium (Gubler, 1997; Gubler, 2002). The need for research and surveillance is often dealt with and many authors have stressed that DF/DHF is still perceived as unimportant and receives little attention despite its social and economic impact being similar to some of the most visible infectious diseases (Meltzer, 1998; Coleman, 2004).

Different mathematical models were proposed. In general, they use compartmental dynamics with Susceptible, Exposed, Infective and Removed for human; and Susceptible and Infective for mosquito. SEIRS models were considered with an evaluation of the impact of ultra-low volume (ULV) insecticide applications on dengue epidemics (Newton, 1992). The values of basis parameters used in simulation by these authors constituted a data source (Table 1) for other authors. A general model with the population of susceptible and infectious human assumed constant and facing only one virus was considered by Esteva and Vargas (1998). These authors also proposed models where the human population was supposed to grow exponentially and to have a constant disease rate (Esteva, 1999), two serotypes of virus and variable human population and the impact of vertical transmission and interrupted feeding on the dynamics of the disease (Esteva, 2000; Esteva 2003).

In previous papers, while pointing out that the idea of two viruses coexisting in the same epidemic is controversial, mathematical models with constant human population ($N_h$) and two different viruses acting at separated intervals of time were considered by the authors (Derouich, 2003; Derouich, 2004). The case of variable human population ($N_h$), was also considered Derouich, 2006). Building on that, the present chapter introduces a compartment of vaccinated people and hence considers a SVIR model.

**FORMULATION OF THE MODEL AND STABILITY ANALYSIS**

**Parameters of the Model**

Let $N_h$ and $N_v$ denote the human and vector population size. In this model death is proportional to the populations size with rate constant $\mu_h$ and we assume a constant $\Lambda_h$ due to births and immigrations. So

$$\frac{dN_h}{dt} = \Lambda_h - \mu_h N_h$$