INTRODUCTION

The preceding chapter is to be viewed as a purely theoretical math intuition and the claim consists in that of the existence of a region within the voxel (the three dimensional pixel) where interpolation is most beneficial because it is meant to produce the least approximation of the true intensity value and this region has been named: “Sub-pixel Efficacy Region” (SRE). An energy function will be defined here as the ratio between the energy of the original image and the energy of the interpolated image. This ratio which is called the Intensity-Curvature Functional, is symbolized by the expression $\Delta E = E_o / E_{IN}$, and it is prone to be studied to reveal its behavior within the voxel, and it is prone to determine the boundary of the Sub-pixel Efficacy Region within the voxel. In this chapter the Intensity-Curvature Functional will be treated for what concerns the trivariate linear interpolation function such to present its original conception. An application of the Intensity-Curvature Functional for the improvement of the trivariate linear interpolation function was however previously reported (Ciulla & Deek, 2005).

$E_o$ is the intensity-curvature term before interpolation and shall be envisioned as the energy of the original signal (image) and $E_{IN}$ is the intensity-curvature term after interpolation and shall be envisioned as the energy of the interpolated signal (image). What is demonstrated in this chapter is that because of interpolation the signal (image) is subject to energy change and the Intensity-Curvature Functional is the measure of such energy change. This chapter presents a mathematical demonstration of this presupposition. $\Delta E$ is a measure of energy change which covers the cases: (i) interpolated and original signal (image) differs in their voxel intensity; or (ii) interpolated and original signal (image) differs in their local second order derivative (curvature); or (iii) interpolated and original signal (image) differs in their voxel intensity and also in their local second order derivative. This is logical to presuppose since the intensity correction inherent to interpolation determines a change in node (voxel) intensity $h(x, y, z)$ with respect to the original value $f(0, 0, 0)$. And also this change of intensity is strictly linked to the change in local curvature of the interpolation function $h(x, y, z)$. 

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The Conception of the Intensity-Curvature Functional

IMAGE ENERGY

The formulation of the interpolation function \( h(x, y, z) \) was given in definition IV of Chapter II. The curvature consists of the second order derivatives of \( h(x, y, z) \): \( \frac{\partial^2}{\partial x \partial y} h(x, y, z) \), \( \frac{\partial^2}{\partial y \partial z} h(x, y, z) \), and \( \frac{\partial^2}{\partial z \partial x} h(x, y, z) \). For a voxel, let the energy of the interpolated image be defined as:

\[
E_{\text{IN}} = E_{\text{IN}}(x, y, z) = E_{\text{IN}}(\Psi_{xy}) + E_{\text{IN}}(\Psi_{zx}) + E_{\text{IN}}(\Psi_{yz}) \tag{1}
\]

where:

\[
E_{\text{IN}}(\Psi_{xy}) = \int_{-x/2}^{x/2} \int_{-y/2}^{y/2} \int_{-z/2}^{z/2} h(x, y, z) \left( \frac{\partial^2}{\partial x \partial y} h(x, y, z) \right) dx \, dy \, dz \tag{2}
\]

\[
E_{\text{IN}}(\Psi_{zx}) = \int_{-x/2}^{x/2} \int_{-y/2}^{y/2} \int_{-z/2}^{z/2} h(x, y, z) \left( \frac{\partial^2}{\partial z \partial x} h(x, y, z) \right) dx \, dy \, dz \tag{3}
\]

\[
E_{\text{IN}}(\Psi_{yz}) = \int_{-x/2}^{x/2} \int_{-y/2}^{y/2} \int_{-z/2}^{z/2} h(x, y, z) \left( \frac{\partial^2}{\partial y \partial z} h(x, y, z) \right) dx \, dy \, dz \tag{4}
\]

For the original image, let the energy be defined as:

\[
E_o = E_o(x, y, z) = E_o(\Psi_{xy}) + E_o(\Psi_{zx}) + E_o(\Psi_{yz}) \tag{5}
\]

where:

\[
E_o(\Psi_{xy}) = \int_{-x/2}^{x/2} \int_{-y/2}^{y/2} \int_{-z/2}^{z/2} f(0, 0, 0) \left( \frac{\partial^2 I}{\partial x \partial y} \right) dx \, dy \, dz \tag{6}
\]

\[
E_o(\Psi_{zx}) = \int_{-x/2}^{x/2} \int_{-y/2}^{y/2} \int_{-z/2}^{z/2} f(0, 0, 0) \left( \frac{\partial^2 I}{\partial z \partial x} \right) dx \, dy \, dz \tag{7}
\]

\[
E_o(\Psi_{yz}) = \int_{-x/2}^{x/2} \int_{-y/2}^{y/2} \int_{-z/2}^{z/2} f(0, 0, 0) \left( \frac{\partial^2 I}{\partial y \partial z} \right) dx \, dy \, dz \tag{8}
\]

DEFINITION I

Let \( h \) be a continuous function, as given by definition IV of Chapter II.