On Generalized Fuzzy Entropy and Fuzzy Divergence Measure with Applications

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ABSTRACT

Entropy in a fuzzy set measures the amount of ambiguity/imprecision presented in the fuzzy set. In this article, the authors introduce a generalized fuzzy entropy measure and demonstrate its effectiveness in Multiple Attribute Decision Making (MADM) and superiority from the point of view of structured linguistic variables. This article also introduces a generalized fuzzy directed divergence and investigates its properties. Further, this article demonstrates the effectiveness of the proposed generalized directed divergence in pattern recognition.

KEYWORDS
Fuzzy Directed Divergence, Fuzzy Entropy, Fuzzy Set (FS), MADM, Pattern recognition

1. INTRODUCTION

The concept of entropy measure is enormously used to measure the amount of uncertainty in a random experiment. Formally, the idea of entropy was proposed by Shannon (1948). In last few decades, enormous work had been reported towards the generalization of probabilistic information measures (For more details, refer to Taneja, 2001). But these measures work only for unambiguous data, i.e., where the data is concerned with a precisely known system.

To deal with ambiguous data/information, Zadeh (1965) introduced fuzzy set theory and incorporated the concept of fuzzy logic to improve upon some shortcomings of binary logic. De Luca and Termini (1972) proposed an axiomatic definition of fuzzy entropy. The entropy of the fuzzy set is considered as the average amount of ambiguity/imprecision associated with that fuzzy set. After De Luca and Termini (1972), many authors proposed the generalization of fuzzy entropy. Kosko (1986) proposed a new fuzzy entropy based on hypercube and distance between fuzzy subset $A$, nearest vertex $A_{near}$ and farthest vertex $A_{fur}$, Hooda (2004), Li and Liu (2008), Bajaj and Hooda (2010) provided parametric generalizations of fuzzy entropy. Verma and Sharma (2011) proposed the generalized exponential fuzzy entropy. Joshi and Kumar (2017b) introduced two parametric exponential fuzzy entropy and many more.

A measure of discrimination between two fuzzy sets is also a significant tool for the study of various engineering problems. Bhandari et al. (1992) proposed the information measure for discrimination between two fuzzy sets. Bhandari and Pal (1993) introduced another discrimination measure which is a special case of entropy proposed by De Luca and Termini (1972). Hooda (2004) provided one parametric generalization of the fuzzy divergence measure of Bhandari et al. (1992). Bhatia and Singh (2012) proposed some generalized fuzzy divergence measures corresponding to some existing probabilistic divergence measures. Bhatia and Singh (2013) had presented an approach using aggregation operators to obtain symmetric divergence and application of fuzzy divergence.
measure in image segmentation. Most of the work related to the generalization of fuzzy entropy and fuzzy divergence measures did not give justification for preference of the generalized versions of fuzzy entropy and fuzzy divergence measures over classical versions of these fuzzy information measures. In the present work, we study one parametric generalization of De Luca and Termini’ (1972) fuzzy entropy and provide justification for preference of the proposed generalized fuzzy entropy. Further, we also study one parametric generalization of Bhandari et al. (1992) divergence measure and establish its superiority in some problems of pattern recognition. Zhu & Li (2016) derived a new entropy formula concerning the amount of information and reliability in intuitionistic fuzzy sets. Fuzzy entropy and divergence measures are useful in various multi-attribute group decision-making methods such as TOPSIS, MOORA, VIKOR, TODIM, etc. Many authors (Chen & Li, 2010; Riu & Li, 2010; Shemshadi et al., 2011; Awasthi & Kannan, 2016; Ren et al., 2016, and Joshi & Kumar, 2018) have studied these methods using fuzzy/intuitionistic fuzzy/interval-valued intuitionistic fuzzy entropy and divergence measures from various viewpoints.

Let $A$ be a given fuzzy set and it has certain amount of imprecision/ambiguity from point of view of an expert. But due to some factors the knowledge of the expert regarding the fuzzy set $A$ may change. There may be involvement of some factor/parameter in enhancing the knowledge of the expert. In such a situation, the amount of ambiguity present in fuzzy set $A$ may not be realistic if it is calculated by non-parametric entropy and hence may lead to some wrong decision. Therefore, generalized (parametric) fuzzy entropies are desirable to deal with such problems. Further, due to adaptive nature of fuzzy concepts all problems of same nature (e.g. decision making, pattern recognition, computer vision, etc.) may not be solved by one generalized fuzzy information (entropy/divergence) measure. Consequently, a new generalized fuzzy information measure is always desirable as per requirements of a problem under consideration. These issues motivated us to develop generalized fuzzy information measures.

The main contributions of this work may be summarized as follows:

1. One-parametric generalized fuzzy entropy is proposed and the monotonic character of the proposed generalized fuzzy entropy is also investigated.
2. The superiority of the proposed generalized fuzzy entropy over some prevalent fuzzy entropy measures for dealing linguistic hedges have been demonstrated through an illustrative example.
3. The application of generalized fuzzy entropy is illustrated in multi-attribute decision-making (MADM). In MADM, the performance of the generalized fuzzy entropy with change in parameter $\alpha$ is found to be consistent.
4. A generalized fuzzy directed divergence based on the generalized fuzzy entropy is proposed and application of the fuzzy directed divergence in pattern recognition is demonstrated through an illustrative example. The performance of the generalized fuzzy directed divergence has also been compared with existing fuzzy divergence measures.

The whole paper is divided into nine sections. Section 2 presents some preliminaries related to our proposed information measures. Section 3 proposes the generalized fuzzy entropy measure of order $\alpha$. Some properties of our proposed measure have also been studied. In Section 4, we study the monotonic character of generalized fuzzy entropy with respect to parameter $\alpha$. Section 5 shows how our proposed fuzzy entropy measure is more efficient than some existing fuzzy entropy measures. In Section 6, we present an algorithm for solving decision-making problems based on our proposed generalized fuzzy entropy. Section 7 provides a generalized fuzzy divergence measure and proof of some of its properties. Section 8 gives the comparative study of the proposed fuzzy divergence measure with some existing fuzzy divergence measures using some illustrative examples. Finally, Section 9 concludes and presents the scope for future work.
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