Fuzzy Lattice Ordered G-modules

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ABSTRACT

The study of mathematics emphasizes precision, accuracy, and perfection, but in many of the real-life situations, people face ambiguity, vagueness, imprecision, etc. Fuzzy set theory and rough set theory are two innovative tools in mathematics which are used for decision-making in vague and uncertain information systems. Fuzzy algebra has a significant role in the current era of mathematical research and it deals with the algebraic concepts and models of fuzzy sets. The study of various ordered algebraic structures like lattice ordered groups, Riesz spaces, etc., are of great importance in algebra. The theory of lattice ordered G-modules is very useful in the study of lattice ordered groups and similar algebraic structures. In this article, the theories of fuzzy sets and lattice ordered G-modules are synchronized in a suitable manner to evolve a novel concept in mathematics i.e., fuzzy lattice ordered G-modules which would pave the way for new researchers in fuzzy mathematics to explore much more in this field.

KEYWORDS

Fuzzy IG-ideal, G-level set, G-support, Lattice Ordered G-module, IG-module Homomorphism

1. INTRODUCTION

The concept of ordering which led to the origin of lattice theory has a significant place in the study of mathematics (Gratzer, 1978). The theory of ordered algebraic structures is an emerging trend in the area of mathematical research. Various lattice ordered algebraic structures like lattice ordered groups, rings fields and Riesz spaces have been studied by many researchers in this field (Birkhoff, 1942; Blyth, 2005; McAlister, 1998; Steinberg, 2010). Besides mathematics, the study of ordered algebraic structures has many applications in quantum logic, analytics and computer science.

The theory of fuzzy sets has proved to be an effective tool to deal with uncertain, vague and incomplete information systems. It has helped to apply mathematical tools to handle ambiguity and imprecision in many situations like data mining, knowledge discovery, approximate reasoning and decision making. A fuzzy set is a function from the universal set $X$ to $[0,1]$ (Zadeh, 1965). Right from the inception of fuzzy sets by L.A Zadeh, many algebraic concepts have been combined with the theory of fuzzy sets to develop a new branch in mathematics namely fuzzy algebra. Various novel algebraic structures like fuzzy groups, fuzzy rings, fuzzy modules, etc., have thus developed in mathematics. The concepts of fuzzy lattices and fuzzy $G$-modules have been studied in (Ajmal & Thomas, 1994) and (Fernandez, 2004) respectively. The invention of fuzzy lattice ordered groups by G.S.V. Satya Saibaba (Satya Saibaba, 2008) is an important milestone in the path of progress of both the theories of fuzzy sets and ordered algebraic structures.

Some researchers have carried out a profound study of crisp algebraic structures in the $L$-fuzzy context where $[0,1]$ is replaced by a lattice structure. The concepts of $L$-fuzzy hyper modules (Yin,
Zhan, Xu, & Wang, 2010), \(L\)-fuzzy poly groups (Yin, Zhan, & Huang, 2011) and “\(-\)-hyper rings (Yin, Davvaz, & Zhan, 2012) are some of the latest research topics in this field.

The authors are mainly interested in the study of \(G\)-modules especially in the fuzzy and rough context. The concepts of rough \(G\)-modules, fuzzy rough \(G\)-modules and rough fuzzy \(G\)-modules have been studied in (Isaac & Ursala, 2017). The notion of lattice ordered \(G\)-modules was introduced in (Ursala & Isaac, 2017). In this paper the authors look forward to combine the theories of fuzzy lattices and fuzzy \(G\)-modules to introduce the new concept of fuzzy lattice ordered \(G\)-modules and discuss some of its properties.

Section 2 recalls some of the basic definitions of lattice ordered \(G\)-modules. In section 3 the concept of fuzzy lattice ordered \(G\)-modules is introduced and some of its properties are studied. Section 4 deals with the properties of the lattice ordered group involved in a fuzzy lattice ordered \(G\)-module. In section 5 the notion of fuzzy lattice ordered \(G\)-ideals is defined and some of its properties are discussed. Section 6 deals with the lattice ordered \(G\)-module homomorphism and some related results. The authors conclude in section 7 with the possible research work that can be done further in the field of fuzzy algebra and representation theory.

2. PRELIMINARIES

In this section, some basic definitions that will be needed in the sequel are given. For crisp algebraic concepts one may refer the books (Curties & Reiner, 1962) and (Fraleigh, 1986) and for more basic definitions related with lattice ordered \(G\)-modules one may refer (Tremblay & Manohar, 1975).

2.1. Definition

(Tremblay & Manohar, 1975) A lattice is a partially ordered set \(\langle L, \leq \rangle\) in which every pair of elements \(a, b \in L\) has a greatest lower bound \(a \wedge b\) (called their meet) and a least upper bound \(a \vee b\) (called their join). Here \(\leq\) is the partial order on the set \(L\).

2.2. Definition

(Tremblay & Manohar, 1975) A lattice is an algebraic system \(\langle L, \wedge, \vee \rangle\) with two binary operations \(\wedge\) and \(\vee\) which are both commutative, associative and satisfy the absorption laws. It has been shown in (Tremblay & Manohar, 1975) that the two definitions are equivalent.

2.3. Definition

(Tremblay & Manohar, 1975) A non-empty subset \(S\) of a lattice \(L\) is called a sublattice of \(L\) if \(a, b \in S \Rightarrow a \wedge b, a \vee b \in S\).

2.4. Definition

(Birkhoff, 1942) A non-empty set \(G\) with a binary operation \(\ast\) and a partial order \(\leq\) is called a lattice ordered group or an \(l\)-group iff

(1) \(\langle G, \ast \rangle\) is a group
(2) \(\langle G, \leq \rangle\) is a lattice
(3) \(g \leq h \Rightarrow a \ast g \ast b \leq a \ast h \ast b\) for all \(a, b, g, h \in G\).

2.5. Example

(Steinberg, 2010) \(\langle Z, + \rangle, \langle Q, + \rangle\) and \(\langle R, + \rangle\) are lattice-ordered groups under the natural ordering.
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