Chapter 11

Bipolar Fuzzy Structure of H–Ideals in BCI–Algebras

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ABSTRACT

In this chapter, the concepts of bipolar fuzzy H-ideals of BCI-algebras are introduced and their natures are investigated. Relations between bipolar fuzzy subalgebras, bipolar fuzzy ideals, and bipolar fuzzy H-ideals are discussed. Conditions for a bipolar fuzzy ideal to be a bipolar fuzzy H-ideal are provided. Some characterization theorems of bipolar fuzzy H-ideals are established. A bipolar fuzzy H-ideal is established by using a finite collection of H-ideals. The authors have shown that if every bipolar fuzzy H-ideal has the finite image, then every descending chain of H-ideals terminates at finite step.

1. INTRODUCTION

Zhang (1994) initiated the concept of bipolar fuzzy sets (BFSs) as a generalization of fuzzy sets (Zadeh, 1965). In fuzzy sets membership degree range is [0,1]. In a BFS membership degree range is increased from the interval [0,1] to the interval [-1,1]. The membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on (0,1] indicate that elements somewhat satisfy the property and the membership degrees on [-1,0] indicate that elements somewhat satisfy the implicit counter-property. This domain has recently motivated new research in several directions (Akram, 2011, Dubois, 2008, Zadrozny & Kacprzyk, 2012, Zhang & Zhang, 2004).


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introduced (intuitionistic) fuzzy translations of (intuitionistic) fuzzy \( H \)-ideals in \( BCK/BCI \)-algebras and investigated their properties in details. The author (together with colleagues) have done lot of works on \( B/BG/G \)-algebras (Bhowmik et al, 2014, Senapati et al. 2012, 2014, 2015a, 2015b, 2015c, 2016) which is related to \( BCK/BCI \)-algebras.

Lee (2009) introduced the notion of bipolar fuzzy subalgebras and ideals in \( BCK/BCI \)-algebras. The concept of bipolar valued fuzzy translation and bipolar valued fuzzy \( S \)-extension of a bipolar valued fuzzy subalgebra in \( BCK/BCI \)-algebra was introduced by Jun et al.(2009). Lee et al. (2011) also extended this study to \( a \)-ideals of \( BCI \)-algebras.

Motivated by this, in this chapter, the notions of bipolar fuzzy \( H \)-ideals (BFHIs) of \( BCI \)-algebras is introduced and their properties are investigated. Relations among bipolar fuzzy subalgebras, bipolar fuzzy ideals and BFHIs are discussed. Conditions for a bipolar fuzzy ideal to be a BFHI are provided. Some characterization theorems of BFHIs are established. A BFHI is established by using a finite collection of \( H \)-ideals.

2. PRELIMINARIES

In this section, some elementary aspects that are necessary for this paper are included.

By a \( BCI \)-algebra we mean an algebra \((X,*,0)\) of type \((2,0)\) satisfying the following axioms for all \( x,y,z \in X \):

1. \(( (x*y)*(x*z) )*(z*y) ) = 0 \]
2. \(( x*(x*y) )*y = 0 \]
3. \( x*x = 0 \]
4. \( x*y = 0 \) and \( y*x = 0 \) imply \( x = y \).

If a \( BCI \)-algebra \( X \) satisfies \( 0*x = 0 \) for all \( x \in X \), then we say that \( X \) is a \( BCK \)-algebra.

We can define a partial ordering \( \leq \) by \( x \leq y \) if and only if \( x*y = 0 \).

A non-empty subset \( S \) of \( X \) is called a subalgebra (Jun, 1993) of \( X \) if \( x*y \in S \) for any \( x,y \in S \).

A non-empty subset \( I \) of \( X \) is called an ideal (Jun, 1993) of \( X \) if it satisfies (I1) \( 0 \in I \) and (I2) \( x*y \in I \) and \( y \in I \) imply \( x \in I \).

A non-empty subset \( I \) of \( X \) is said to be a \( H \)-ideal (Khalid & Ahmed, 1999) of \( X \) if it satisfies (I1) and (I3) \( x*y*z \in I \) and \( y \in I \) imply \( x*x \in I \) for all \( x,y,z \in X \).

A \( BCI \)-algebra is said to be associative (Hu & Iseki, 1980) if \( (x*y)*z = x*(y*z) \) for all \( x,y,z \in X \).

**Definition 2.1** (Zadeh, 1965) Let \( X \) be the collection of objects denoted generally by \( x \), then a fuzzy set \( A \) in \( X \) is defined as \( A = \{ \langle x, \lambda_A(x) \rangle : x \in X \} \), where \( \lambda_A(x) \) is called the membership value of \( x \) in \( A \) and \( 0 \leq \lambda_A(x) \leq 1 \).

**Definition 2.2** (Jun, 1993) A fuzzy set \( A = \{ \langle x, \lambda_A(x) \rangle : x \in X \} \) in \( X \) is called a fuzzy subalgebra of \( X \) if it satisfies the inequality \( \lambda_A(x*y) \geq \min\{\lambda_A(x), \lambda_A(y)\} \) for all \( x,y \in X \).