Chapter 12

Lukaswize Triple-Valued Intuitionistic Fuzzy BCK/BCI-Subalgebras

Chiranjibe Jana
Vidyasagar University, India

Karping Shum
Yunnan University, China

ABSTRACT

The authors studied the notion of $(\alpha,\beta)$-intuitionistic fuzzy BCK/BCI-subalgebras by applying the Lukasiewicz 3-valued implication operator, where $\alpha,\beta \in \{\in, \in\land, \in\lor\}$ for $\alpha \neq \in\land$. In this chapter, an intuitionistic fuzzy set $A$ is an $(\in, \in)$ (or $(\in\land, \in)$ or $(\in, \in\lor)$)-intuitionistic fuzzy subalgebras if and only if for any $p \in (0,1]$ (or $p \in (0,0.5]$ or $p \in (0.5,1]$), then $A_p$ is a fuzzy subalgebras of $X$ respectively. Next, the authors defined intuitionistic fuzzy subalgebras with thresholds $(s,t)$ and then provided intuitionistic fuzzy subalgebras with thresholds $(0,1)$ (or $(0,0.5)$ or $(0.5,1)$) by the concept of quasi-coincidence of fuzzy point respectively. Also, $A$ is an intuitionistic fuzzy subalgebras with thresholds $(s,t)$ if and only if for any $p \in (s,t]$, then cut set $A_p$ is a fuzzy subalgebras.

1 INTRODUCTION

The role of logic in mathematics and computer science is two-fold which acts as a tool for applications in theoretical areas as well as a technical areas for laying the foundations. Thus, non-classical logic including many-valued logic and fuzzy logic takes the advantage of the classical logic (Boms and Mack, 1975) to handle information with various facets of uncertainty (Zadeh, 2005, 2008) such as fuzziness, randomness and so on. Presently, non-classical logic has become a formal and useful tool for computer science which can be used to deal with uncertain information and fuzzy information. The semantical systems of non-classical logic systems have produced various logical algebras. In 1966, Imai and Iseki

DOI: 10.4018/978-1-7998-0190-0.ch012
Lukaswize Triple-Valued Intuitionistic Fuzzy BCK/BCI-Subalgebras

(1966) introduced a notions of logical algebras called BCK/BCI.algebras based on the generalization of the concept of set-theoretic difference and propositional calculus.

The concept of fuzzy sets was proposed by Zadeh in his tremendous paper (Zadeh, 1965), provided a natural mathematical framework for generalizing some notions of classical algebraic structures. After the introduction of fuzzy sets many researchers conducted the researches on the generalizations of fuzzy sets that have successful applications in computer science, artificial intelligence, control engineering, expert, robotics, automat theory, finite state machine, graph theory, logics and many branches of pure and applied mathematics. In 1971. Rosenfeld (1971) introduced the notion of fuzzy group. Since then many enormous papers have been published in fuzzy algebra. Mordeson and Malik introduced fuzzy subring and fuzzy subalgebra in ring theory. Jana et al. (2015), Bej and Pal (2015) have been done lot of works on fuzzy BCK/BCI/G.algebras. Based on this concept Bhakat and Das (1996) introduced a new generalization of fuzzy subgroups. Then this concept extended to BCK/BCI.algebras by many researchers in (Jun, 2005, 2007; Ma et al., 2011, 2012; Ma et al. (2009)). In 1999. the intuitionistic fuzzy sets was proposed by Atanassov (1999), a nobel generalization of fuzzy sets. Then different types of fuzzy setting have been conducted on intuitionistic fuzzy subgroups by many researchers in (Jana et al., 2016; Jun & Kim, 2000; Larimi, 2013; Yuan, 2010; Zhan & Tan, 2004). Though fuzzy sets and intuitionistic fuzzy sets have a applications in different theoretical study as well as real life applications. Implication operator with intuitionistic fuzzy sets have a applications in logical point of view. These points are enough to motivated us and best of our knowledge there is no work available on 3.valued intuitionistic fuzzy BCK. algebra. For this reason, we developed theoretical study of 3.valued intuitionistic fuzzy subalgebras of BCK/BCI.algebras based on Lukasiewicz 3.valued implication operator.

In this paper, applying the Lukasiewicz implication operators, the notions of (α,β).intuitionistic fuzzy subalgebras of BCK/BCI.algebras has been defined by using the grades of a fuzzy point $x_a$ on intuitionistic fuzzy sets $A$. Also, $(\in,\in), (\in\land q), \text{ and } (\in\lor q)$ intuitionistic fuzzy subalgebra of $X$ are investigated and define intuitionistic fuzzy subalgebra with thresholds $(s,t)$ of $X$.

2 PRELIMINARIES

In this section, some elementary aspects that are necessary for this paper are included.

By a BCI.algebra we mean an algebra $(X,*,0)$ of type (2,0) satisfying the following axioms for all $x,y,z\in X$.

(C1) $((x*y)*(x*z))*(z*y)=0$
(C2) $(x*(x*y))*y=0$
(C3) $x*x=0$
(C4) $x*y=0$ and $y*x=0$ imply $x=y$.

The partial ordering is defined as $x\leq y$ if and only if $x*y=0$.

If a BCI.algebra $X$ satisfies $0*x=0$ for all $x\in X$ then $X$ is a BCK.algebra. Any BCI.algebra $X$ satisfies the following axioms for all $x,y,z\in X$.

(C5) $(x*y)*z=(x*z)*y$
(C6) $((x*z)*(y*z))*(x*y)=0$