Chapter 16
Some Types of Ideals in QI–Algebras

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ABSTRACT

In this chapter, ideals of QI-algebra are considered. Given a subset of a right distributive QI-algebra, the smallest ideal containing it is constructed. Also, the notions of implicative ideal, fantastic ideal, and normal ideal in a right distributive QI-algebra are introduced, and the authors proved that these notions are equivalent.

1. INTRODUCTION

BCK-algebras and BCI-algebras were introduced by Imai and Iseki(1966), Iseki and Tanaka(1978) and Iseki(1980). Since their introduction, several generalizations of BCK-algebras were introduced and extensively studied by many researchers. Abbott (1967) introduced a concept of an implication algebra in the sake to formalize the logical connective implication in the classical propositional logic. Recently, Saeid et al. (2017) introduced the concept of a BI-algebra as a generalization of (dual) implication algebra and studied its properties. Bandaru(2017) introduced the concept of a QI-algebra as a generalization of BI-algebra and studied the concept of ideals, congruences in a QI-algebra. Also given connection between ideals and congruence kernels whenever a QI-algebra is right distributive. In this paper, we introduce the notions of implicative ideal, fantastic ideal and normal ideal in a right distributive QI-algebra and we show that these notions are equivalent.

DOI: 10.4018/978-1-7998-0190-0.ch016
2. PRELIMINARIES

First, we recall certain definitions and properties from Saeid et al. (2017), Abbott (1967) & Chen et al. (1995) that are required in the paper.

**Definition 2.1** (Iseki, 1980) A BCI-algebra is an algebra \((X, *, 0)\) of type \((2,0)\) satisfying the following conditions:

1. \((x*y)*(x*z)\leq (z*y)\)
2. \(x*(x*y)\leq y\)
3. \(x\leq x\)
4. \(x\leq y\) and \(y\leq y\) imply \(x=y\)
5. \(x\leq 0\) implies \(x=0\)

where \(x\leq y\) is defined by \(x*y=0\).

If (5) is replaced by (6) \(0\leq x\), then the algebra is called a BCK-algebra. It is known that every BCK-algebra is a BCI-algebra but not conversely. A BCK-algebra satisfying the property \(x*(y*x)=x\) for all \(x, y\in X\) is called an implicative BCK-algebra.

Several generalizations of a BCK-algebra, in the form of definitions, one can see in the paper (Saeid et al., 2017).

**Definition 2.2** (Abbott, 1967) A groupoid \((X, *)\) is called an implication algebra if it satisfies the following identities:

(a) \((x*y)*x=x\)
(b) \((x*y)*y=(y*x)*x\)
(c) \(X*(y*z)=y*(x*z)\)

for all \(x, y, z\in X\).

**Definition 2.3** (Abbott, 1967) Let \((X, *)\) be an implication algebra and binary operation \(\circ\) on \(X\) be defined by \(x*y=y\circ x\). Then \((X, \circ)\) is said to be a dual implication algebra. In fact, the axioms of that are as follows:

(a) \(x \circ (y \circ x) = x\)
(b) \(x \circ (x \circ y) = y \circ (y \circ x)\)
(c) \((x \circ y) \circ z = (x \circ z) \circ y\)

for all \(x, y, z\in X\).

Chen & Oliveira (1995) proved that in any implication algebra \((X, *)\) the identity \(x*y=y*x\) holds for all \(x, y\in X\). We denote the identity \(x*x=y*y\) by the constant 0. The notion of BI-algebras comes from the (dual) implication algebra.