Chapter VI
Graph Theoretic Techniques
in the Analysis of Uniquely Localizable Sensor Networks

Bill Jackson
University of London, UK

Tibor Jordán
Eötvös University, Hungary

ABSTRACT

In the network localization problem the goal is to determine the location of all nodes by using only partial information on the pairwise distances (and by computing the exact location of some nodes, called anchors). The network is said to be uniquely localizable if there is a unique set of locations consistent with the given data. Recent results from graph theory and combinatorial rigidity made it possible to characterize uniquely localizable networks in two dimensions. Based on these developments, extensions, related optimization problems, algorithms, and constructions also became tractable. This chapter gives a detailed survey of these new results from the graph theorist’s viewpoint.

INTRODUCTION

In the network localization problem the locations of some nodes (called anchors) of a network as well as the distances between some pairs of nodes are known, and the goal is to determine the location of all nodes. This is one of the fundamental algorithmic problems in the theory of wireless sensor networks and has been the focus of a number of recent research articles and survey papers, see for example (Aspnes et al., 2007; Eren et al., 2004; Mao et al., 2007; So & Ye, 2007).

A natural additional question is whether a solution to the localization problem is unique. The network, with the given locations and distances, is said to be uniquely localizable if there is a unique set
of locations consistent with the given data. The unique localizability of a two-dimensional network, whose nodes are ‘in generic position’, can be characterized by using results from graph rigidity theory. In this case unique localizability depends only on the combinatorial properties of the network: it is determined completely by the *distance graph* of the network and the set of anchors, or equivalently, by the *grounded graph* of the network and the number of anchors. The vertices of the distance and grounded graph correspond to the nodes of the network. In both graphs two vertices are connected by an edge if the corresponding distance is explicitly known. In the grounded graph we have additional edges: all pairs of vertices corresponding to anchor nodes are adjacent. The grounded graph represents all known distances, since the distance between two anchors can be obtained from their locations. Before stating the basic observation about unique localizability we need some additional terminology. It is convenient to investigate localization problems with distance information by using frameworks, the central objects of rigidity theory.

A $d$-dimensional *framework* (also called *geometric graph* or *formation*) is a pair $(G, p)$, where $G = (V, E)$ is a graph and $p$ is a map from $V$ to $\mathbb{R}^d$. We consider the framework to be a straight line realization of $G$ in $\mathbb{R}^d$. Two frameworks $(G, p)$ and $(G, q)$ are *equivalent* if corresponding edges have the same lengths, that is, if $||p(u) − p(v)|| = ||q(u) − q(v)||$ holds for all pairs $u, v$ with $uv \in E$, where $||.||$ denotes the Euclidean norm in $\mathbb{R}^d$. Frameworks $(G, p), (G, q)$ are *congruent* if $||p(u) − p(v)|| = ||q(u) − q(v)||$ holds for all pairs $u, v$ with $u, v \in V$. This is the same as saying that $(G, q)$ can be obtained from $(G, p)$ by an isometry. We shall say that $(G, p)$ is *globally rigid*, or that $(G, p)$ is a *unique realization* of $G$, if every framework which is equivalent to $(G, p)$ is congruent to $(G, p)$, see Figure 1.

The next observation shows that the theory of globally rigid frameworks is the mathematical background which is needed to investigate the unique localizability of networks.

**Theorem 1.** (Aspnes et al., 2006; So & Ye, 2007) Let $N$ be a network in $\mathbb{R}^d$ consisting of $m$ anchors located at positions $p_1, ..., p_m$ and $n−m$ ordinary nodes located at $p_{n+1}, ..., p_n$. Suppose that there are at least $d+1$ anchors in general position. Let $G$ be the grounded graph of $N$ and let $p = (p_1, ..., p_n)$. Then the network is uniquely localizable if and only if $(G, p)$ is globally rigid.

We shall give a survey of the current status of the theory of globally rigid graphs and frameworks, focusing on the most relevant cases of two and three-dimensional frameworks, but stating results for higher dimensions, wherever possible. We will assume that the reader is familiar with the basic terms of graph theory. Readers who are not can find them in the Appendix.

*Figure 1. Two realizations of the same graph $G$ in $\mathbb{R}^2$: $F_1$ is globally rigid; $F_2$ is not since we can obtain a realization of $G$ which is equivalent but not congruent to $F_2$ by reflecting $p_2$ in the line through $p_1, p_3, p_5$.***
Related Content

Early Work on UV Sensitive Solid Photocathodes for Gaseous Detectors
www.igi-global.com/chapter/early-work-on-uv-sensitive-solid-photocathodes-for-gaseous-detectors/153738?camid=4v1a

CsI–RICH Detectors
www.igi-global.com/chapter/csirich-detectors/153747?camid=4v1a

Biometrics: In Search of a Foolproof Solution
www.igi-global.com/chapter/biometrics-search-foolproof-solution/23818?camid=4v1a

Voluntary Blink Controlled Communication Protocol for Bed-Ridden Patients
www.igi-global.com/chapter/voluntary-blink-controlled-communication-protocol-for-bed-ridden-patients/162383?camid=4v1a