INTRODUCTION

The purpose of this chapter is to illustrate by means of examples the construction of the analysis and synthesis filter banks with the use of FIR and IIR two-channel filter banks as the basic building blocks. In Chapter VIII, we have discussed the design and properties of several types of complementary filter pairs, and in Chapters IX and X we have shown how those filter pairs are used in the synthesis of digital filters with sharp spectral constraints. In this chapter, we demonstrate the application of the complementary filter pairs as two-channel filter banks used to decompose the original signal into two channel signals and to reconstruct the original signal from the channel signals. Signal decomposition is referred to as the signal analysis, whereas the signal reconstruction is referred to as the signal synthesis. Thereby, the filter bank used for the signal decomposition is called the analysis filter bank, and the bank used for signal reconstruction is called the synthesis filter bank.

The two-channel filter bank is usually composed of a pair of lowpass and highpass halfband filters, which satisfy some complementary properties. The bandwidth that occupies each of two channel signals is a half of the original signal bandwidth. Hence, the channel signals can be processed with the sampling rate which is a half of the original signal sampling rate. At the output of the analysis bank, the channel signals are down-sampled-by-two and then processed at the lower sampling rate. For the signal reconstruction, each of two channel signals has to be up-sampled-by-two first, and then fed into the synthesis bank.

The sampling rate alteration in the two-channel filter bank causes the unwanted effects: the down-sampling produces aliasing, and the up-sampling produces imaging. The essential feature of the two-
Examples of Multirate Filter Banks

channel filter bank is that the aliasing produced in the analysis side can be compensated in the synthesis side. This is achieved by choosing the proper combination of filters in the analysis and synthesis banks. The elimination of aliasing opens the possibility of the perfect (and nearly perfect) reconstruction of the original signal. The perfect reconstruction means that the signal at the output of the cascade connection of the analysis and synthesis bank is a delayed replica of the original input signal. Constructing perfect reconstruction and nearly perfect reconstruction analysis/synthesis filter banks is an unbounded area of research.

An important and widely used application of the two-channel filter banks is the construction of multichannel filter banks based on the tree-structures where the two-channel filter bank is used as a building block. In this way, a multilevel multichannel filter bank can be obtained with either uniform or nonuniform separation between the channels. The two-channel filter banks are particularly useful in generating octave filter banks.

Depending on applications, the filter bank can be requested to provide frequency-selective separation between the channels, or to preserve the original waveform of the signal. The example applications of the frequency-selective filter banks are audio and telecommunication applications. The importance of preserving the original waveform is related with the images. In the case of the discrete-time wavelet banks, the frequency-selectivity is less important. The main goal is to preserve the waveform of the signal.

The purpose of this chapter is to illustrate by means of MATLAB examples the signal analysis and synthesis based on the two-channel filter banks. We give first a brief review of the properties of the two-channel filter banks with the conditions for aliasing elimination. We discuss the perfect reconstruction and nearly perfect reconstruction properties and show the solutions based on FIR and IIR QMF banks and the orthogonal two-channel filter banks. In the sequel, the tree-structured multichannel filter banks are considered. The process of signal decomposition and reconstruction is illustrated by means of examples.

TWO-CHANNEL FILTER BANKS

The block diagram representing the analysis/synthesis two-channel filter bank with the processing unit between the analysis and synthesis parts is shown in Figure 12.1.

In the analysis bank, the original signal \(x[n]\) is filtered using the lowpass/highpass filter pair \([H_0(z), H_1(z)]\), and the lowpass and highpass channel signals \(x_0[n]\) and \(x_1[n]\) are obtained. Therefore, their z-transforms \(X_0(z)\) and \(X_1(z)\) are given by

\[
X_0(z) = H_0(z)X(z), \text{ and } X_1(z) = H_1(z)X(z).
\] (12.1)

Figure 12.2 illustrates typical magnitude frequency responses of \(H_0(z)\) and \(H_1(z)\).

The spectra of the filtered signals \(x_0[n]\) and \(x_1[n]\) occupy a half of the baseband of the original signal \(x[n]\), and according to this, \(x_0[n]\) and \(x_1[n]\) can be further processed at the half of the input sampling rate. The filtered signals \(x_0[n]\) and \(x_1[n]\) are then down-sampled by-a-factor-of-two, and subband signals \(v_0[n]\) and \(v_1[n]\) are obtained. If the sampling rate at the input is \(F_0\), the subband signal components \(v_0[n]\) and \(v_1[n]\) are sampled at the rate \(F_0/2\).