Chapter 2
Symmetries of the Degeneracy of the Vertebrate Mitochondrial Genetic Code in the Matrix Form

ABSTRACT
Symmetries of the degeneracy of the vertebrate mitochondrial genetic code in the mosaic matrix form of its presentation are described in this chapter. The initial black-and-white genomatrix of this code is reformed into a new mosaic matrix when internal positions in all triplets are permuted simultaneously. It is revealed unexpectedly that for all six variants of positional permutations in triplets (1-2-3, 2-3-1, 3-1-2, 1-3-2, 2-1-3, 3-2-1) the appropriate genetic matrices possess symmetrical mosaics of the code degeneracy. Moreover the six appropriate mosaic matrices in their binary presentation have the general non-trivial property of their “tetra-reproduction,” which can be utilized in particular for mathematical modeling of the phenomenon of the tetra-division of gametal cells in meiosis. Mutual interchanges of the genetic letters A, C, G, U in the genomatrices lead to new mosaic genomatrices, which possess similar symmetrical and tetra-reproduction properties as well.

INTRODUCTION AND BACKGROUND
Chapter 1 described the construction of genomatrices of the Kronecker family, including the genomatrix $P^{CAUG}_{123} = [C A; U G]^3$, which contain 64 triplets in the well-ordered form. But how are amino acids and stop-codons, which are encoded by these triplets, disposed in this genomatrix? Does the genetic code possess any features which may give the symmetrical character for this genomatrix? Such questions are investigated in this chapter. Really, the degeneracy of the genetic code has lead to a symmetrical black-and-white mosaic of the genomatrix in the case of the vertebrate mitochondrial genetic code, which is the most symmetrical dialect of the genetic code.

DOI: 10.4018/978-1-60566-124-7.ch002
Symmetries of the Degeneracy of the Vertebrate Mitochondrial Genetic Code

By analogy of the theory of digital signals, where permutations of signal elements play significant role, we study two kinds of permutations of elements of the genetic code, which transform initial mosaic genomatrices into new mosaic genomatrices. The first of these kinds of permutations is permutations of three positions inside all triplets: 1-2-3, 2-3-1, 3-1-2, 1-3-2, 2-1-3, 3-2-1. The second kind is mutual interchanges of the genetic letters A, C, G, U. Both of these kinds lead unexpectedly to such new genomatrices, which possess symmetrical black-and-white mosaics and the binary forms of which possess the mathematical property of tetra-self-reproducing. This chapter sets out results of these investigations.

The main objectives of this chapter are the following:

1. In-depth study of matrix symmetries of the degeneracy of the vertebrate mitochondrial genetic code in the matrix form of its presentation;
2. Investigations of reforming these matrix symmetries under some kinds of permutations of elements of the genetic code;
3. Demonstrating new phenomenological materials in the field of matrix genetics to develop algebraic models of the genetic code.

PECULIARITIES OF DEGENERACY OF THE GENETIC CODE

Modern science knows many dialects of the genetic code, data about which are shown on the NCBI’s website http://www.ncbi.nlm.nih.gov/Taxonomy/Utils/wprintgc.cgi. According to general traditions, theory of symmetry studies initially those natural objects which possess the most symmetrical character, and then it constructs a theory for cases of violations of this symmetry in other kindred objects. Correspondingly the authors of this book investigate initially the vertebrate mitochondrial genetic code which is the most symmetrical code among dialects of the genetic code. One can also note that some authors consider this dialect not only as the most “perfect” but also as the most ancient dialect (Frank-Kamenetskiy, 1988) while the last aspect is a debatable one. The vertebrate mitochondrial code is used as a basic dialect in some other mathematical works where a presentation of the 64 triplets exists in a form of square tables (Dragovich & Dragovich, 2006, 2007; Khrennikov & Kozyrev, 2007). Figure 1 shows the correspondence between the set of 64 triplets and the set of 20 amino acids with stop-signals (Stop) of protein synthesis in this code.

The set of 64 triplets contains such 16 subfamilies of triplets, every one of which contains 4 triplets with the same two letters on the first positions of each triplet (an example of such subsets is the case of the four triplets CAC, CAA, CAU, CAG with the same two letters CA on their first positions). We shall name such subfamilies as the subfamilies of NN-triplets. In the case of the vertebrate mitochondrial code, the set of these 16 subfamilies of NN-triplets is divided into two equal subsets from the viewpoint of degeneration properties of the code (Figure 1). The first subset contains 8 subfamilies of so called “two-position” NN-triplets, a coding value of which is independent of a letter on their third position. An example of such subfamilies is the four triplets CGC, CGA, CGU, CGC, all of which encode the same amino acid Arg, though they have different letters on their third position. All members of such subfamilies of NN-triplets are marked by black color in the genomatrix \([\text{C A; U G}]\) on the Figure 2.

The second subset contains 8 subfamilies of “three-position” NN-triplets, a coding value of which depends on a letter on their third position. An example of such subfamilies is the four triplets CAC, CAA, CAU, CAC, two of which (CAC, CAU) encode the amino acid His and other two (CAA, CAG) encode...