Chapter 6
The Genetic Code, Hadamard Matrices, Noise Immunity, and Quantum Computers

ABSTRACT
This chapter continues an analysis of the degeneracy of the vertebrate mitochondrial genetic code in the matrix form of its presentation, which possesses the symmetrical black-and-white mosaic. Taking into account a symmetry breakdown in molecular compositions of the four letters of the genetic alphabet, the connection of this matrix form of the genetic code with a Hadamard (8x8)-matrix is discovered. Hadamard matrices are one of the most famous and the most important kinds of matrices in the theory of discrete signals processing and in spectral analysis. The special U-algorithm of transformation of the symbolic genetic matrix \([C A; U G]^{(3)}\) into the appropriate Hadamard matrix is demonstrated. This algorithm is based on the molecular parameters of the letters A, C, G, U/T of the genetic alphabet. In addition, the analogical relations is shown between Hadamard matrices and other symmetrical forms of genetic matrices, which are produced from the symmetrical genomatrix \([C A; U G]^{(3)}\) by permutations of positions inside triplets. Many new questions arise due to the described fact of the connection of the genetic matrices with Hadamard matrices. Some of them are discussed here, including questions about an importance of amino-group NH\(_2\) in molecular-genetic systems, and about possible relations with the theory of quantum computers, where Hadamard gates are utilized. A new possible answer is proposed to the fundamental question concerning reasons for the existence of four letters in the genetic alphabet. Some thoughts about cyclic codes and a principle of molecular economy in genetic informatics are presented as well.

INTRODUCTION AND BACKGROUND
We continue to investigate connections of the genetic matrices with matrix formalisms of the theory of discrete signals processing. One of the most famous and the most important kinds of matrices in this

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Figure 1. The family of Hadamard matrices $H(2^k)$ based on the Kronecker product

$$H(2) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} ; \quad H(4) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$H(2^k) = \begin{bmatrix} H(2^{k-1}) & H(2^{k-1}) \\ -H(2^{k-1}) & H(2^{k-1}) \end{bmatrix}$$

theory are the so called Hadamard matrices. These matrices are used also in many other fields due to their advantageous properties: in error-correcting codes such as the Reed-Muller code; in spectral analysis; in multi-channels spectrometers with Hadamard transformations; in quantum computers with Hadamard gates (or logical operators), in quantum mechanics as unitary operators, etc.

Does any natural connection exist between the genetic matrices, which were described in previous chapters, and Hadamard matrices? This question should be investigated especially because a possible positive answer to it may lead to many significant consequences and new thoughts about structures of the genetic code. This chapter demonstrates the existence of such a connection and analyzes some questions related to it.

A huge number of scientific publications are devoted to Hadamard matrices. These matrices give effective opportunities for information processing.

By definition a Hadamard matrix of dimension \( "n" \) is the \((nxn)\)-matrix \( H(n) \) with elements \("+1"\) and \("-1"\). It satisfies the condition \( H(n)\ast H(n)^\top = n\ast I_n \), where \( H(n)^\top \) is the transposed matrix and \( I_n \) is the \((nxn)\)-identity matrix. The Hadamard matrices of dimension \( 2^k \) are given by the recursive formula \( H(2^k) = H(2)^{(k)} = H(2)\otimes H(2^{k-1}) \) for \( 2 \leq k \in N \), where \( \otimes \) denotes the Kronecker (or tensor) product, \( (k) \) means the Kronecker exponentiation, \( k \) and \( N \) are integers, \( H(2) \) is demonstrated in Figure 1.

Rows of a Hadamard matrix are mutually orthogonal. It means that every two different rows in a Hadamard matrix represent two perpendicular vectors, a scalar product of which is equal to 0. The element \("-1"\) can be disposed in any of four positions in the Hadamard matrix \( H(2) \). Such matrices are used in many fields due to their advantageous properties: in error-correcting codes such as the Reed-Muller code; in spectral analysis and multi-channel spectrometers with Hadamard transformations; in quantum computers with Hadamard gates, etc. It was discovered unexpectedly that Hadamard matrices reflect essential peculiarities of molecular genetic systems (Petoukhov, 2005, 2006, 2008a-d).

Normalized Hadamard \((2x2)\)-matrices are matrices of rotation on \( 45^\circ \) or \( 135^\circ \) depending on an arrangement of signs of its individual elements. A Kronecker product of two Hadamard matrices is a Hadamard matrix as well. A permutation of any columns or rows of a Hadamard matrix leads to a new Hadamard matrix.

Hadamard matrices and their Kronecker powers are used widely in spectral methods of analysis and processing of discrete signals and in quantum computers. A transform of a vector \( \bar{a} \) by means of a Hadamard matrix \( H \) gives the vector \( \bar{u} = H\ast \bar{a} \), which is named Hadamard spectrum. A greater analogy between Hadamard transforms and Fourier transforms exists (Ahmed & Rao, 1975). In particular the fast Hadamard transform exists in parallel with the fast Fourier transform. The whole class of multichannel “spectrometers with Hadamard transforms” is known (Tolmachev, 1976), where the principle of tape
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