Chapter 7

Genomatrices and the Genetic Octet Yin–Yang–Algebras

ABSTRACT

Algebraic properties of the genetic code are analyzed. The investigations of the genetic code on the basis of matrix approaches (“matrix genetics”) are described. The degeneracy of the vertebrate mitochondrial genetic code is reflected in the black-and-white mosaic of the (8*8)-matrix of 64 triplets, 20 amino acids, and stop-signals. The special algorithm, which is based on features of genetic molecules, exists to transform the mosaic genomatrix into the matrices, which are members of the special 8-dimensional algebra. Main mathematical properties of this genetic algebra and its relations with other algebras are analyzed together with some important consequences from the adequate algebraic models of the genetic code. Elements of new “genovector calculation” and ideas of “genetic mechanics” are discussed. The revealed fact of the relation between the genetic code and these genetic algebras, which define new multi-dimensional numeric systems, is discussed in connection with the famous idea by Pythagoras: “All things are numbers.” Simultaneously, these genetic algebras can be utilized as the algebras of genetic operators in biological organisms. The described results are related to the problem of algebraization of bioinformatics. They draw attention to the question: what is life from the viewpoint of algebra?

INTRODUCTION AND BACKGROUND

Does the genetic system possess its own algebra? Why is it important to study the question about the proper algebra of the genetic code? These questions are analyzed in this chapter first of all. Modern science knows that different natural systems can possess their own individual geometries and their own individual algebras (see, for example, the book (Kline, 1980)). The example of Hamilton, who wasted 10 years in
his attempts to solve the task of description of transformations of 3D space by means of 3-dimensional algebras without a success, is very demonstrative one. This example implies that if a scientist does not guess right what type of algebras are adequate for the natural system which is investigated by him he can waste many years without any result in analogy with Hamilton. One can add that geometrical and physical-geometrical properties of separate natural systems (including laws of conservations, theories of oscillations and waves, theories of potentials and fields, etc.) can depend on the type of algebras which are adequate for them.

Matrix genetics have important analogues with matrix forms of presentations of hypercomplex numbers. Investigations of these analogues have led to adequate models of the genetic code in forms of multi-dimensional numeric systems, which are connected with appropriate multi-dimensional algebras. Such algebraic models of the genetic code put forward many new ideas and thoughts about interrelations among genetic elements and about relations of structures of the genetic code with many other biological, physical, information and mathematical structures.

Does the genetic system possess its own algebra? Why is it important to study the question about the proper algebra of the genetic code? To get answers on these questions and to understand their importance, the following background is useful.

The notion of “number” is the main notion of mathematics. In accordance with the famous thesis, the complexity of civilization is reflected in the complexity of the numbers which are utilized by the civilization. “Number is one of the most fundamental concepts not only in mathematics, but also in all natural sciences. Perhaps, it is the more primary concept than such global categories, as time, space, substance or a field.” (Pavlov, 2004)

After the establishment of real numbers in the history of the development of the notion of “number”, complex numbers \( x_0 + i x_1 \) have appeared. These 2-dimensional numbers have played the role of the magic tool for development of theories and calculations in problems of heat, light, sounds, fluctuations, elasticity, gravitation, magnetism, electricity, current of liquids, and quantum-mechanical phenomena. It seems that modern atomic stations, airplanes, rockets and many other things would not exist without knowledge of complex numbers because the appropriate physical theories are based on these numbers.

C. Gauss, J. Argand and C. Wessel have demonstrated that a plane with its properties fits 2-dimensional complex numbers. W. Hamilton has proved that the properties of our 3-dimensional physical space fit mathematical properties of the special quaternions. Hamilton’s quaternions have played the significant role in the history of mathematical natural sciences as well. For example, the classical vector calculation is deduced from the theory of these quaternions. This chapter will show that the genetic code is connected with a special 8-dimensional numeric system, which is defined by the appropriate 8-dimensional algebra.

The notion “algebra”, which we use in our book, has two main senses. According to the first sense, which is famous more widely, the algebra is the whole section of mathematics involving mathematical operations with mathematical symbols. According to the second sense, which is utilized in this book, algebra is a mathematical object with certain properties or, better to say, arithmetic of multidimensional numbers.

By definition in the frame of this second sense, algebra \( A \) with its dimension “\( n \)” over a field \( P \) is a set of expressions \( x_0 i_0 + x_1 i_1 + x_2 i_2 + \ldots + x_{n-1} i_{n-1} \) (where \( x_0, x_1, \ldots, x_{n-1} \) belong \( P \); \( i_0, i_1, \ldots, i_{n-1} \) are basic elements of vectors, which fit such expressions). This set is provided with the operation of multiplication by elements “\( k \)” from the field \( P \) to determine the formula \( k (x_0 i_0 + x_1 i_1 + x_2 i_2 + \ldots + x_{n-1} i_{n-1}) = k x_0 i_0 + k x_1 i_1 + k x_2 i_2 + \ldots + k x_{n-1} i_{n-1} \). This set is provided with the following operation of addition