Chapter 12
A Global Optimization Approach to Solve Multi-Aircraft Routing Problems

S.P. Wilson
Numerical Optimisation Centre, University of Hertfordshire, UK

M.C. Bartholomew-Biggs
Numerical Optimisation Centre, University of Hertfordshire, UK

S.C. Parkhurst
Numerical Optimisation Centre, University of Hertfordshire, UK

ABSTRACT
This chapter describes the formulation and solution of a multi-aircraft routing problem which is posed as a global optimization calculation. The chapter extends previous work (involving a single aircraft using two dimensions) which established that the algorithm DIRECT is a suitable solution technique. The present work considers a number of ways of dealing with multiple routes using different problem decompositions. A further enhancement is the introduction of altitude to the problems so that full three-dimensional routes can be produced. Illustrative numerical results are presented involving up to three aircraft and including examples which feature routes over real-life terrain data.

INTRODUCTION
The aircraft routing problem involves finding paths (in two or three dimensions) from a specified start point to a specified destination (possibly also passing through some rendezvous points on the way). Routes are composed of straight line segments joining intermediate “waypoints” and the problem is to position these waypoints so that the resulting path avoids certain “hard” constraints (such as geographical features and “no-fly zones”) and has low exposure to “soft” constraints (such as risks from military missile threats or low fuel reserves). The optimality of a route is based on minimizing a “cost” composed of a weighted sum of penalties for constraint violations together with similar penalties for lateness at rendezvous points, flying excessive distances, using extreme manoeuvres et cetera. More discussion about operational aspects of aircraft routing can be found in (Hewitt & Broatch,
A Global Optimization Approach

1992) and (Hewitt & Martin, 1998) which also describe a heuristic routing algorithm.

Other papers which discuss different versions of the aircraft routing problem include (Carlyle et al., 2007; Chen et al., 1995; Karczewski, 2007; Zabarankin et al., 2006).

This chapter is an extension of earlier work (Bartholomew-Biggs, Parkhurst & Wilson, 2003, both papers) in which the aircraft routing problem was tackled using a number of global optimization direct search techniques applied to a basic two-dimensional problem which we shall refer to as the Simplified Route Model (SRM). It was found that the deterministic DIRECT algorithm (Jones, Perttunen & Stuckman, 1993) appeared better suited to this problem than the random-sampling and tabu-search techniques of TSHJ (Al-Sultan & Al-Fawzan, 1997) and ECTS (Chelouah & Siarry, 2000).

Our previous work has been concerned with finding routes for a single aircraft. This chapter extends the SRM cost function both to handle multiple routes and also to introduce altitude to the problem so that full three-dimensional routes can be produced. We present results using both the SRM and also a Realistic Route Model (RRM) (Wilson, 2003), involving actual terrain data.

The SRM test results given in this chapter were obtained from a Pentium 3 (500Mhz) PC running on a Linux platform. Hardware details for the RRM model cannot be disclosed. All of the software was implemented in C/C++.

THE SIMPLIFIED ROUTE MODEL (SRM) ROUTE COST FUNCTION

We first summarise the main features of the Simplified Route Model; more details can be found in (Wilson, 2003). In the two-dimensional case we can attempt to find the ground plan of a route, avoiding a number of obstacles that we shall henceforth call “threats”. A route will be defined by its (given) start and end points and by a number of intermediate waypoints. The co-ordinates of these waypoints will be optimization variables; and we assume that the flight path follows straight lines between them. We shall now describe a Simplified Route Model (SRM) route cost calculation.

Suppose that the distance flown is to be as short as possible, subject to suitable avoidance of the threats. Then, for any choice of waypoints, we first calculate the Euclidean length of the corresponding route, denoted by \( L \), say. We can write \( L = \sum l_j \) where \( l_j \) is the length of the “leg” between the \( j \)-th and \( (j+1) \)-th waypoint. We then determine how much of the route passes through threats. If the route passes through the \( i \)-th threat for a distance \( L_i \) then the “cost” of the route can be expressed as

\[
C = L + \sum_i \rho_i L_i^p
\]  

where \( \rho_i \) is a penalty parameter associated with the \( i \)-th threat and \( p \) is an integer exponent to be discussed below. Our aim will be to choose the waypoint co-ordinates so as to minimize \( C \). The balance between flight path length and threat penetration will depend upon the choice of the parameters \( \rho_i \). If the \( i \)-th threat is a physical obstacle then \( \rho_i \) must be large to ensure that the solution does not attempt to pass through it; but if threat \( i \) represents some risky but not impossible region then a more moderate value of \( \rho_i \) may be appropriate because it will allow a shorter route which makes an acceptably brief incursion into some danger area. Hence, in (1), the length of leg \( j \) that lies inside threat \( i \) is defined as \( L_{ij} = \sum l_{ji} \).

In the SRM examples used in this chapter we shall use circular threats; but in general we may need to deal with no-fly zones and geographical obstacles with irregular boundaries. Therefore we must calculate threat violations using a sampling process along each leg of a route. We suppose that it is possible to determine whether a point \((x, y)\) is inside or outside a threat but that no explicit