Chapter 9

Statistical Analysis of Computational Intelligence Algorithms on a Multi-Objective Filter Design Problem

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ABSTRACT

This chapter presents the application of a comprehensive statistical analysis for both algorithmic performance comparison and optimal parameter estimation on a multi-objective digital signal processing problem. The problem of designing optimum digital finite impulse response (FIR) filters with the simultaneous approximation of the filter magnitude and phase is posed as a multi-objective optimization problem. Several computational-intelligence-based algorithms for solving this particular optimization problem are presented: genetic algorithms (GA), particle swarm optimization (PSO) and simulated annealing (SA) with multi-objective scalarization methods. Algorithms with Pareto sampling methods, namely non-dominated sorting genetic algorithm II (NSGA-II) and multi-objective simulated annealing (MOSA) are also applied as a way of dealing with multi-objective optimization. Instead of using a process of trial and error, a statistical exploratory analysis is used to estimate optimal parameters. A comprehensive statistical comparison of the applied algorithms is addressed, which indicates a particularly strong performance of NSGA-II and pure GA with weighting scalarization.

INTRODUCTION

This chapter addresses the application of several computational-intelligence-based algorithms for solving a nonlinear multi-objective digital signal processing problem. Also, this work develops multi-objective extensions to existing single-objective statistical exploratory analysis. The developed methodology is
primarily intended to obtain an estimate of best parameter values for the adaptive algorithms and also to effectively compare performance. In this regard, a binary quality indicator will be used for comparison of the Pareto front approximations obtained.

The rest of the chapter is organized as follows: in the next section, the digital signal processing problem is described – more specifically, it is a multi-objective optimization problem on digital filter design. Optimization theory to better analyze the problem at hand is subsequently discussed. Following which, the optimization algorithms that will be applied to the signal processing problem are addressed. After that, the statistical exploratory analysis and its extensions to the multi-objective case are developed. The results on the digital signal processing problem are then presented, and finally, the last section presents conclusions about the work.

DIGITAL FILTER DESIGN THEORY

Introduction

In a strict sense, the term frequency selective filter suggests a system that passes certain frequency component and totally rejects all others, but in a broader context, any system that modifies certain frequencies relative to others is also classified as a filter (Oppenheim & Schafer, 1999).

A digital filter can be uniquely characterized on the time-domain by its impulse response $h(n)$. Alternatively, a digital filter can also be uniquely characterized on the frequency domain by its frequency response $H(e^{j\omega})$, which is also the DTFT (Discrete-Time Fourier Transform) of the sequence $h(n)$ (Madisetti & Williams, 1999).

Digital filters can be classified into two main classes:

- **FIR (Finite Impulse Response)**, also known as non-recursive filters.
- **IIR (Infinite Impulse Response)**, also known as recursive filters.

For the FIR filters, $h(n)$ is nonzero for a finite number of samples while on the IIR filters, $h(n)$ has an infinite number of nonzero samples. Additionally, particularly for the FIR filters, $h(n)$ sequence samples are also known as the filter coefficients. For the IIR filters, the filter coefficients include feedback terms in the filter difference equation (Diniz, da Silva, & Lima, 2004).

Digital filter design has been extensively addressed within the last 25 years. The design and realization of digital filters involve a blend of theory, applications, and technologies (Madisetti & Williams, 1999). On most applications, the filter design specifications are given on the frequency domain by the filter desired frequency response $D(e^{j\omega})$. It is important to note that $D(e^{j\omega})$ is in general a complex-valued function. This work focuses on the simultaneous arbitrary approximation, in the optimum sense, of both the magnitude and the phase of the response.

Given that the frequency response of a digital filter is always periodic on the frequency variable $\omega$ with period equal to $2\pi$, the filter design specifications need only be defined on a period of the frequency variable, usually in the frequency region $[-\pi, \pi]$. Besides that, when the filter frequency response is conjugate symmetric $D^*(\omega) = D(e^{-j\omega})$, it suffices to specify the frequency response on positive frequencies $[0,2\pi]$ (Madisetti & Williams, 1999). This symmetric-conjugate condition on the filter frequency