Chapter 13
The Evolution of the Hermite Transform in Biomedical Applications

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ABSTRACT

The set of Hermite functions has been widely applied in biomedical signal processing, especially in medical data processing and more recently in clinical decision making. This article will review the advantages of the continuous Hermite functions, discuss some specific biomedical applications to electrocardiograms and other signals, as well as applications to medical image processing. The main goal of this article is to explain a new method of generating a discrete version of the continuous Hermite functions to biomedical signals and images that are digital. The process is computationally simple, and provides a set of vectors that share many of the properties of the continuous Hermite functions that are otherwise lost in the discretization process. Specific applications of this Discrete Hermite Transform (DHmT) are provided that include (i) monitoring ischemia from electrocardiograms, (ii) artifact removal from electroencephalogram, and (iii) ultra wide band wireless monitoring of respiration rate. This DHmT holds much promise in biomedical signal and image processing.

INTRODUCTION

Biomedical Engineering is a rapidly advancing area, where new scientific and engineering concepts are being applied to enhance clinical care by bringing about advancement in diagnostic and treatment methodologies. In the fast paced world, with an increasing number of patients and conditions, there is a need to develop intelligent medical technologies that assist the physicians in automating tasks and make clinical decision making an effortless endeavor. The complex nature of biomedical data makes it harder to interpret the different characteristics of the signal that are not explicit and hence the need to translate them to ‘compact’ representations is the first step in the automation process. Two important factors to consider are (1) The closeness these representations have to the original data, and
The computational efficiency in calculating these representations in real-time. The former has an impact on the medical decision, whereas the latter in the timing of the decision. Both are critical to clinical care.

An important feature of biomedical signals is the changing frequency content over a wide band that is localized in time. A simple example would be a long term ECG signal with an initial normal ‘horizontal’ ST segment elevated due to infarction and depressed later with an inverted T post-infarction. Conventional Fourier transform techniques were inadequate for representing such signals, the primary reason being that sinusoidal basis functions were inefficient in compactly representing a complex biomedical signal, i.e. they required a large number of coefficients. Therefore, attention shifted to time-frequency analysis techniques such as Wigner and Gabor transforms. These, however, suffered setbacks because of fixed resolution in the time-frequency plane, i.e. they were better off with narrow band signals. The introduction of Wavelets was a huge breakthrough since a perfect choice of a wavelet could efficiently represent a signal at any spatio-temporal resolution with a fewer number of coefficients.

Continuous Hermite transforms have gained popularity recently because of their inherent advantage over wavelets that includes Gaussianity and orthogonality properties. A discrete version of the Hermite transform is essential since all biomedical data collection, storage and analysis are becoming more and more digital. The continuous case loses its properties when discretized using conventional sampling techniques and the process could often be too time consuming to be realized in real-time.

The objective of this article is to present a new method of generating discrete Hermite transforms (DHmT) that is fast to be realized in real-time and efficiently preserves the orthogonality properties. The DHmT presented here satisfies both the criteria that are listed above that are critical to clinical care. In this chapter, the focus is on the application involving biomedical signals and images, although other topics are touched upon briefly. In section II, we review the basic properties of the continuous Hermite transform, followed by a thorough literature survey on the evolution of continuous Hermite transforms in biomedical engineering. In section III, a new method of generating discrete Hermite functions using the centered Fourier matrix and symmetric Tridiagonal matrix is presented. In section IV, several biomedical applications of this new transform that includes (i) monitoring ischemia from electrocardiograms, (ii) artifact removal from electroencephalogram, and (iii) ultra wide band wireless monitoring of respiration rate are described in detail.

BACKGROUND

Properties of the Continuous Hermite Transform

The Hermite functions were named after Charles Hermite, a French Mathematician (1822 – 1901) who discovered them. The Hermite functions are formed by a product of the Hermite polynomials with the Gaussian function.

\[ u_n(t) = H_n(t) e^{-t^2/2} \text{ for } n \geq 0 \]  

(1)

Hermite Polynomials \( H_n(t) \), \( n=0,1,2,3,... \) form a complete orthogonal set on the interval \( -\infty < t < \infty \). The Hermite polynomials, using the Rodrigues Formula (a formula for producing a series of expressions by repeated differentiation of another function) is given by

\[ H_n(t) = (-1)^n e^{-t^2} \frac{d^n}{dt^n} e^{t^2} \quad \text{for } n = 0,1,2,3,..., -\infty \leq t \leq \infty \]  

(2)