Chapter 1
Dynamical Systems and Their Chaotic Properties

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ABSTRACT

This chapter describes the fundamental concepts of continuous-time chaotic dynamical systems so as to make the book self-contained and readable by fairly unfamiliar people with using simple chaotic systems and understanding their chaotic properties, but also by experts who desire to refine their knowledge regarding chaos theory. It will also define the sense in which a chaotic system is suitable for real-world applications such as encryption.

1. INTRODUCTION

In this chapter the basic characteristics of several systems of different types will be described theoretically and examined numerically. The types of dynamical systems are limited to ordinary and delay differential equations and the examples given are the three famous pendulum, Ikeda and Mackey-Glass equations. The chaos quantifiers used in chaos theory namely the attractor dimension and the metric entropy together with the Lyapunov exponents are described with a brief text regarding the way of their numerical evaluation. Furthermore, linearized stability analysis of a delay equation is derived in detail and lastly a system of coupled delay equations that describes a semiconductor laser subjected to optical feedback is presented and investigated since it is widely used for chaos encryption. The purpose of this chapter is to familiarize the reader with the basic principles of chaos and how the latter may be treated numerically.

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1.1 Dynamical Systems

A dynamical system is one whose state changes in time. If the changes are determined by specific rules, rather than being random, the system is said to be deterministic, and in this chapter only deterministic systems will be considered. The changes can occur at discrete time steps or continuously. This chapter will be concerned only with continuous-time, deterministic, dynamical systems, since they best approximate the real world. Ordinary and time-delayed differential equations will be described together with prototypical elegant famous examples together with their chaotic properties. The most obvious examples of dynamical systems are those that involve something moving through space like a planet orbiting the Sun or a pendulum swinging back and forth. But dynamical systems can also be more abstract, such as money flowing though the economy, information propagating across the Internet, or disease moving though the population. We should mention that discrete-time systems have already been extensively explored, in part because they are more computationally tractable and therefore will not be investigated and mentioned herein.

Calculation of the motion of astronomical bodies is one of earliest problems solved by scientists, and the calculations are some the most precise in all of science, allowing, for example, the prediction of eclipses many years in advance both in time and space. Contrast this strong predictability with the difficulty of predicting even a few days in advance whether the sky will be clear enough to observe that eclipse using extremely detailed models and vast computational resources. Therein lays the difference between regular and chaotic dynamics. It is therefore prudent to pose the question: How one can measure how complex a system is? The answer lies on the chaos quantifiers, namely the metric entropy, the attractor dimension and generally the Lyapunov exponents. These measures will be defined next and will be calculated for the examples of the chaotic systems. But first we will describe some basic concepts from dynamical systems theory.

1.2 Chaos and Strange Attractors: Background Material from Dynamical Systems Theory

A basic example of an ordinary differential equation (or else termed as flow) is the pendulum (Matthews, 2005) that consists of a mass \( m \) on the end of massless, rod of length \( L \). In the pendulum, one time-dependent variable is the angle \( x \) (in radians) that the pendulum makes from the vertical. At each instant, there is a force equal to \( -\sin(x) \) pulling the mass back to its equilibrium position at \( x = 0 \). Newton’s second law leads to the following system of motion equations:

\[
\begin{align*}
\frac{dx}{dt} &= u \\
\frac{du}{dt} &= -\sin(x)
\end{align*}
\]  

(1.1)

where \( u \) is the angular velocity. In some sense this system has one degree of freedom that spans its motion in two different directions \( x \) and \( u \) and there is a unique direction and amplitude of the motion given by the vector whose components are \( \frac{dx}{dt} \) and \( \frac{dy}{dt} \). The simultaneous motion of all the points in this space resembles a flowing fluid, and hence systems such as Eq. (1.1) are usually called flows.
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