Some Remarks on the Concept of Approximations from the View of Knowledge Engineering

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ABSTRACT

The concepts of approximations in granular computing (GrC) vs. rough set theory (RS) are examined. Examples are constructed to contrast their differences in the Global GrC Model (2nd GrC Model), which, in pre-GrC term, is called partial coverings. Mathematically speaking, RS-approximations are “sub-base” based, while GrC-approximations are “base” based, where “sub-base” and “base” are two concepts in topological spaces. From the view of knowledge engineering, its meaning in RS-approximations is rather obscure, while in GrC, it is the concept of knowledge approximations.

Keywords: Approximation, Granular Computing, Information Technology, Knowledge, Rough Set

INTRODUCTION

Approximation is a serious concept in rough set theory (RS); it defines the rough sets. While in granular computing (GrC), it can be considered from three semantic views: Knowledge Engineering (KE), Uncertainty mathematics, and how-to-compute/solve-it [5]. Each view will have its own theory. In this paper, we will focus on KE view. This paper is a continuous effort that was initiated in Lin (2006b) and Barot and Lin (2008).

RS-APPROXIMATIONS IN (INFINITE) UNIVERSE

The approximation theory of RS is well known. For preciseness, we will recall the notion here. Let U be a classical set, called the universe. Let \( \beta \) be a partition, namely, a family of subsets, called equivalence classes, that are mutually disjoint and their union is the whole universe U.
Then the pair \((U, \beta)\) is called approximation space in RS. Pawlak introduced following two definitions. Observe that Pawlak focus on finite universe. However we allow \(U\) to be infinite. Let \(X\) be an arbitrary subset of the universe \(U\).

**Definition (RS) 1** Let \(E\) be an arbitrary equivalence class of \(R\).

1. Upper approximation:
   \[
   U[X] = \bigcup \{ E | E \cap X \neq \emptyset \}
   \]
2. Lower approximation:
   \[
   L[X] = \bigcup \{ E | E \subseteq X \}
   \]

This definition is the formal form of the intuitive upper and lower approximations.

**Definition (RS) 2** Let \(p\) be an arbitrary element of \(U\).

1. Closure
   \[
   C[X] = \{ p | \forall E, \text{if } p \in E, \text{ then } E \cap X \neq \emptyset \}; \text{ note that } C[X] \text{ is a closed set in the sense of topological spaces.}
   \]
2. Interior
   \[
   I[X] = \{ p | \exists E \text{ such that } p \in E \text{ and } E \subseteq X \}.
   \]

In RS community, the previous definitions are directed generalized to Covering \(Cov\) by interpreting \(E\) as member of \(Cov\).

### COUNTER INTUITIVE PHENOMENA

In this section, we present some Counter Intuitive phenomena of approximations. The first example was generated to answer some questions raised in a conversation with Tian Yang, Guangming Lang, Jing Hao from Hunan University.

**Example 1.** Let the universe \(U\) be the real line. Let us consider the collection \(COV\) of all open half lines, namely, the sets of the following form \(\{ u | u < a \}\) and \(\{ u | a < u \}\) for \(a \in U\). These half lines form a sub-base of the usual topology in real line; Here the “usual topology “ is a technical term referring to the topology of commonly known set.
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