Non Linear Dynamical Systems and Chaos Synchronization

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ABSTRACT

In this paper, the authors study chaos synchronization of chaotic systems, which can exhibit a two scroll attractor for different parameter values via linear feedback control. First, chaos synchronization of three dimensional systems is studied and ‘generalized non-linear dynamical systems’ are analyzed. The considered synchronization criterion consists of identical drive and response systems coupled with linear state error variables. As a consequence, the authors have proposed some theorems for synchronization. This paper features sufficient synchronization criteria for the linear coupled generalized non-linear dynamical systems obtained in an explicit algebraic form and the new synchronization criteria for some typical chaotic systems. Finally, the optimized criteria are applied to explain the Rossler system.

Keywords: Chaos Synchronization, Dynamical Systems, Lyapunov Function, Negative Definite Matrix, Positive Definite Matrix, Rossler System

INTRODUCTION

In 1963, Lorenz simplified the Navier-Stokes equations of modeling weather forecasting and discovered sensitivity dependence on initial conditions in a set of three ordinary differential equations. Li and Yorke first presented the name “chaos” in the sense of “period three implies chaos”. Chaos embodies three important principles: extreme sensitivity to initial conditions, cause and effect being not proportional, as well as nonlinearity. Since the discovery of Lorenz system, more chaotic systems have been constructed such as Rossler system, hyperchaotic Rossler system, Chua’s circuit, Henon attractor, logistic map, Chen system, generalized Lorenz system etc.

Chaos has been intensively studied in the last three decades. Almost every nonlinear system in chaotic state is very sensitive to its initial conditions and often practically exhibits irregular behavior. Thus, one might be wise enough to avoid and eliminate such behavior. Chaos control (Chen & Dong, 1998; Liu & Caraballo, 2006; Yan & Yu, 2007; Yassen, 2003) and synchronization (Afraimovich, Cordonet, & Rulkov, 2002; Li, Lu, & Wu, 2005; Lian, Chiang, & Chiu, 2001; Rulkov, Afraimovich, & Lewis, 2001; Rulkov, 2001; Rulkov & Lewis, 2001; Bai & Lonngren, 2000; Heagy, Carroll, & Pecora, 1994; Itoh, Yang, & Chua, 2001; Pecora & Carroll, 1999; Tanaka & Wang, 1998; Xiaofeng & Guanrong, 2007) has attracted a

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great deal of attention from various fields. Over the last decades, many methods and techniques have been developed, such as OGY method, active control (Bai et al., 2000), feedback and non feedback control (Khan & Singh, 2008; Yu, 1997; Just, Bernard, Ostheimer, Reibold, & Benner, 1997; Pyragas, 1992; Toa, 2006; Yan & Yu, 2007; Yassen, 2005), Impulsive synchronization (Wang, Guan, & Xiao, 2004), adaptive feedback (Huber, 1989; Sun, Tian, Jiang, & Xu, 2007; Yan & Yu, 2007) etc. Recently, linear state error feedback synchronization (Xiaofeng et al., 2007) has provoked a renewal of interest within the context of chaotic dynamical system. Some sufficient conditions of the chaos synchronization approach are derived based on Lyapunov stabilization arguments. According to these conditions and the idea of the adaptive feedback synchronization, we design a new adaptive feedback control method. Compared with some common chaos synchronization methods such as linear feedback control, linear state error feedback control, and feedback and non-feedback control, adaptive feedback control has its characteristic that it is easy to operate in practice, when controlling non-linear dynamical chaotic systems.

FORMULATION OF NONLINEAR DYNAMICAL SYSTEMS

Consider the drive system:

\[ \dot{x}_d[t] = F(t, x_d) , \]  
\[ \dot{x}_r[t] = F(t, x_r) , \]  
and response system:

\[ \dot{y}_r[t] = F(t, y_r) + u[t] \]

where the subscripts “d” and “r” stand for the drive system and response system, respectively. If we denote

\[ x_d = (x_{d1}, x_{d2}, x_{d3}, ..., x_{d1}, ..., x_{dn})^T \]

and \( y_r = (y_{r1}, y_{r2}, y_{r3}, ..., y_{r1}, ..., y_{rn})^T \) as drive system variable and response system variables respectively and

\[ F(t, x_d) = Ax_d + f(t, x_d) \]  
and

\[ F(t, y_r) = Ay_r + f(t, y_r) \]

where \( F : R^n \rightarrow R^n \) is a function that consists of linear and non linear functions \( Ax_d \), \( Ay_r \) and \( f(t, x_d), f(t, y_r) \) respectively. \( u[t] = (u_1[t], u_2[t], u_3[t], ..., u_k[t], ..., u_n[t])^T \) is a control function of time \( t \) and the state variables \( (x_d, y_r) \).

On the basis of all explained terms we can restate (2.1) and (2.2) as follows:

\[ \dot{x}_{d1}[t] = a_{11}x_{d1}[t] + a_{12}x_{d2}[t] + ...... + a_{1k}x_{d}[t] + ...... + a_{1n}x_{nd}[t] + f_1(t, x_d[t]) \]
\[ \dot{x}_{d2}[t] = a_{21}x_{d1}[t] + a_{22}x_{d2}[t] + ...... + a_{2k}x_{d}[t] + ...... + a_{2n}x_{nd}[t] + f_2(t, x_d[t]) \]
\[ \dot{x}_{d3}[t] = a_{31}x_{d1}[t] + a_{32}x_{d2}[t] + ...... + a_{3k}x_{d}[t] + ...... + a_{3n}x_{nd}[t] + f_3(t, x_d[t]) \]
\[ \dot{x}_{dn}[t] = a_{n1}x_{d1}[t] + a_{n2}x_{d2}[t] + ...... + a_{nk}x_{d}[t] + ...... + a_{nn}x_{nd}[t] + f_n(t, x_d[t]) \]

as a drive system and

\[ \dot{y}_{r1}[t] = a_{11}y_{r1}[t] + a_{12}y_{r2}[t] + ...... + a_{1k}y_{r}[t] + ...... + a_{1n}y_{r}[t] + f_1(t, y_r[t]) + u_1[t] \]
\[ \dot{y}_{r2}[t] = a_{21}y_{r1}[t] + a_{22}y_{r2}[t] + ...... + a_{2k}y_{r}[t] + ...... + a_{2n}y_{r}[t] + f_2(t, y_r[t]) + u_2[t] \]
\[ \dot{y}_{rn}[t] = a_{n1}y_{r1}[t] + a_{n2}y_{r2}[t] + ...... + a_{nk}y_{r}[t] + ...... + a_{nn}y_{r}[t] + f_n(t, y_r[t]) + u_n[t] \]
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